ELECTRON & MUON-NEUTRINO SCATTERING

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 7, Exercise 7.8.

In section 7.5, L&P calculate the scattering cross section for the reaction

$$e^{-}(p)+\nu_{\mu}(k)\rightarrow\mu^{-}\left(p'\right)+\nu_{e}\left(k'\right)\tag{1}$$

This is an inelastic interaction in which the final particles are different from the initial particles. The Lagrangian for this process is their equation 7.121:

$$\mathscr{L}_{\text{int}}=\frac{G_F}{\sqrt{2}}\overline{\psi}_{(\nu_e)}\gamma^{\lambda}\left(1-\gamma_5\right)\psi_{(e)}\overline{\psi}_{(\mu)}\left(1-\gamma_5\right)\psi_{\nu_{\mu}}\tag{2}$$

The calculation of the Feynman amplitude from this Lagrangian uses the same techniques as those used for muon decay into 3 particles, which L&P calculate in detail in their section 7.2.2. They then complete the calculation in both the centre of momentum (CM) and lab frames in sections 7.5.1 and 7.5.2. In the CM frame the result is

$$\frac{d\sigma}{d\Omega}=\frac{G_F^2}{4\pi^2}\frac{\left(s-m_{\mu}^2\right)^2}{s}\tag{3}$$

where s is the square of the total energy, so that

$$s=\left(p_0+k_0\right)^2\tag{4}$$

Since this doesn't depend on scattering angle, the total cross section is just 4π times this:

$$\sigma=\frac{G_F^2}{\pi}\frac{\left(s-m_{\mu}^2\right)^2}{s}\tag{5}$$

In the lab frame, we have

$$\frac{d\sigma}{d\Omega_2}=\frac{G_F^2}{4\pi^2}\left(\frac{m_e+2m_e\omega-m_{\mu}^2}{m_e+\omega-\omega\cos\theta_2}\right)^2\tag{6}$$

where ω is the energy of the neutrino in the lab frame. Note that ω is *not* the same as the energy of the neutrino in the CM frame, even though it still travels at the speed of light in both frames.

We now have a dependence on the scattering angle θ_2, but the integral is fairly easy to do. The integral over the azimuthal angle ϕ is just 2π. We get

$$\sigma = \frac{G_F^2}{4\pi^2}\left(m_e^2+2m_e\omega-m_\mu^2\right)^2\int\frac{d\Omega}{\left(m_e+\omega-\omega\cos\theta_2\right)^2} \tag{7}$$

$$= \frac{G_F^2}{4\pi^2}\left(m_e^2+2m_e\omega-m_\mu^2\right)^2\int\frac{\sin\left(\theta_2\right)d\theta_2\,d\phi}{\left(m_e+\omega-\omega\cos\theta_2\right)^2} \tag{8}$$

$$= \frac{G_F^2}{2\pi}\left(m_e^2+2m_e\omega-m_\mu^2\right)^2\int_0^\pi\frac{\sin\left(\theta_2\right)d\theta_2}{\left(m_e+\omega-\omega\cos\theta_2\right)^2} \tag{9}$$

$$= \frac{G_F^2}{2\pi}\left(m_e^2+2m_e\omega-m_\mu^2\right)^2\left.\frac{-1}{\omega\left(m_e+\omega-\omega\cos\theta_2\right)}\right|_0^\pi \tag{10}$$

$$= \frac{G_F^2}{\pi}\frac{\left(m_e^2+2m_e\omega-m_\mu^2\right)^2}{m_e^2+2m_e\omega} \tag{11}$$

where I used Maple to do the algebra in the last step.

Comparing this with the result 5 in the CM frame, we see that the lab result is the same as the CM result with

$$s = m_e^2+2m_e\omega \tag{12}$$

We can see why this makes sense by doing a transformation from the lab frame to the CM frame. To do this, we need the Lorentz transformations for energy and momentum, which are

$$E = \gamma_v\left(E'+vp'\right) \tag{13}$$

$$p = \gamma_v\left(p'+vE'\right) \tag{14}$$

In these equations, v is the velocity of the unprimed frame relative to the primed frame and

$$\gamma_v \equiv \frac{1}{\sqrt{1-v^2}} \tag{15}$$

In our case, v is the velocity of the CM frame relative to the rest frame of the electron. If we picture the neutrino coming in from the $-x$ direction and hitting the electron at $x=0$, then the CM frame is moving to the left, so we can give it a velocity of $-v$. In the lab frame, the energy of the electron is

m_e since it is at rest. As we're assuming the neutrino is massless, its energy is equal to its momentum, with both being ω.

In the CM frame, the magnitudes of the momenta are equal. The momentum of the electron is $\gamma_v m_e v$ and of the neutrino is $\gamma_v(\omega - v\omega)$ so we have

$$\gamma_v m_e v = \gamma_v(\omega - v\omega) \tag{16}$$

or, solving for v:

$$v = \frac{\omega}{m_e + \omega} \tag{17}$$

The energies in this frame are, for the electron (E_v) and neutrino (ω_v):

$$E_v = \gamma_v m_e \tag{18}$$

$$\omega_v = \gamma_v \omega (1 - v) \tag{19}$$

$$= \gamma_v \omega \frac{m\omega}{m + \omega} \tag{20}$$

The parameter s is therefore

$$s = (E_v + \omega_v)^2 \tag{21}$$

$$= \gamma_v^2 \left[\frac{m_e(m_e + \omega) + m_e\omega}{m_e + \omega}\right]^2 \tag{22}$$

$$= \gamma_v^2 \frac{\left(m_e^2 + m_e\omega\right)^2}{(m_e + \omega)^2} \tag{23}$$

To simplify this, we work out

$$\gamma_v^2 = \frac{1}{1 - \frac{\omega^2}{(m_e+\omega)^2}} \tag{24}$$

$$= \frac{(m_e + \omega)^2}{m_e^2 + 2m_e\omega} \tag{25}$$

So we get

$$s = m_e^2 + m_e\omega \tag{26}$$

which is the same as 12.

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