ELECTRON & MUON-NEUTRINO SCATTERING - MINIMUM NEUTRINO ENERGY

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Post date: 28 Aug 2018.

References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 7, Exercise 7.9.

We want to find the minimum energy of the muon neutrino in order that the reaction

$$e^{-}(p)+\nu_{\mu}(k)\rightarrow\mu^{-}\left(p^{\prime}\right)+\nu_{e}\left(k^{\prime}\right) \tag{1}$$

can occur in the rest frame of the electron. Since a muon is considerably more massive than an electron, the incoming neutrino must carry enough energy to allow this extra mass to be created, as well as satisfying the conservation of energy and momentum.

In their discussion of 2-particle scattering, L&P derive the condition on the energy, which arises from a delta function, given in their equation 7.115:

$$E_{2}^{\prime}=\frac{m_{1}^{2}+2m_{1}E_{2}-m_{1}^{\prime2}}{2\left(m_{1}+E_{2}-E_{2}\cos\theta_{2}\right)} \tag{2}$$

where in our case, m_1 is the rest mass of the target particle (the electron), m_1' is the rest mass of the outgoing particle (the muon) and E_2 is the energy of the incoming scattering particle (the muon neutrino). E_2' is the energy of the outgoing massless particle (the electron neutrino) and θ_2 is the scattering angle. Since the electron neutrino is assumed to be massless, its minimum energy is zero. Thus to find the minimum energy of the incoming neutrino we find the value of E_2 that makes $E_2'=0$. This requires only that the numerator in 2 is zero, so

$$E_{2}=\frac{m_{1}^{\prime2}-m_{1}^{2}}{2m_{1}} \tag{3}$$

$$=\frac{m_{\mu}^{2}-m_{e}^{2}}{2m_{e}} \tag{4}$$

Using

$$m_\mu = 106 \text{ MeV} \tag{5}$$
$$m_e = 0.511 \text{ MeV} \tag{6}$$

this gives a value of

$$E_{\nu_\mu}(\text{min}) = 10994 \text{ MeV} \tag{7}$$

Notice that this result is independent of the scattering angle. It requires considerably more energy that that required to create a muon at rest, because the incoming momentum must be conserved in the motion of the two outgoing particles.