BREMSSTRAHLUNG: CROSS-SECTION SUMMED OVER POLARIZATION

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 9, Exercise 9.9.

Bremsstrahlung is the radiation produced by a charge that decelerates in an external magnetic field. In their section 9.10, L&P derive the cross section for the emission of low-energy photons, which is, in lowest order

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_0\frac{\alpha}{(2\pi)^2}\int_{\text{IR}}\frac{d^3k}{\omega}\left[\frac{p'\cdot\epsilon}{p'\cdot k}-\frac{p\cdot\epsilon}{p\cdot k}\right]^2+\ldots \tag{1}$$

where ϵ is the polarization vector, k is the 4-momentum of the photon and $\omega=|\mathbf{k}|$ is the energy of the photon. The subscript IR on the integral indicates that the integral is done over small values (the infra-red region) of ω. The volume element d^3k indicates an integration over the spatial component of the 4-momentum k, so contains a factor of $|\mathbf{k}|^2=\omega^2$. The k in the denominator of the squared factor contributes a factor of ω^{-2}, so the overall integrand contains a factor of ω^{-1} which means the integral is of order $\ln\omega$ and thus diverges as $\omega\to 0$. This is the infra-red divergence.

We can sum 1 over the transverse polarizations using the earlier formula

$$\sum_{r=1,2}\epsilon_r^{\mu}(k)\,\epsilon_r^{\nu}(k)=-g^{\mu\nu}-\frac{k^{\mu}k^{\nu}-(k\cdot n)(n^{\mu}k^{\nu}+k^{\mu}n^{\nu})}{(k\cdot n)^2} \tag{2}$$

$$=-g^{\mu\nu}-\frac{k^{\mu}k^{\nu}}{\mathfrak{k}^2}+\frac{n^{\mu}k^{\nu}+k^{\mu}n^{\nu}}{\mathfrak{k}} \tag{3}$$

where

$$k\cdot n=\mathfrak{k}=\omega=|\mathbf{k}| \tag{4}$$

Multiplying out the bracket in 1 and summing, we have

$$\sum_{r=1,2}\left[\frac{p'\cdot\epsilon}{p'\cdot k}-\frac{p\cdot\epsilon}{p\cdot k}\right]^2=\left[\frac{p'_\mu p'_\nu}{(p'\cdot k)^2}+\frac{p_\mu p_\nu}{(p\cdot k)^2}-2\frac{p'_\mu p_\nu}{(p'\cdot k)(p\cdot k)}\right]\sum_{r=1,2}\epsilon_{r,\mu}\epsilon_{r,\nu} \tag{5}$$

$$=\left[\frac{p'_\mu p'_\nu}{(p'\cdot k)^2}+\frac{p_\mu p_\nu}{(p\cdot k)^2}-2\frac{p'_\mu p_\nu}{(p'\cdot k)(p\cdot k)}\right]\times\left[-g^{\mu\nu}-\frac{k^\mu k^\nu}{\not{k}^2}+\frac{n^\mu k^\nu+k^\mu n^\nu}{\not{k}}\right] \tag{6}$$

$$=-\left(\frac{p'}{p'\cdot k}\right)^2-\left(\frac{p}{p\cdot k}\right)^2+2\frac{p'\cdot p}{(p'\cdot k)(p\cdot k)}-\frac{1}{\not{k}^2}\left(\frac{(p'\cdot k)^2}{(p'\cdot k)^2}+\frac{(p\cdot k)^2}{(p\cdot k)^2}-2\frac{(p'\cdot k)(p\cdot k)}{(p'\cdot k)(p\cdot k)}\right)+\frac{1}{\not{k}}\left(2\frac{(p'\cdot k)(p'\cdot n)}{(p'\cdot k)^2}+2\frac{(p\cdot k)(p\cdot n)}{(p\cdot k)^2}-2\frac{(p\cdot k)(p'\cdot n)+(p'\cdot k)(p\cdot n)}{(p'\cdot k)(p\cdot k)}\right) \tag{7}$$

The last two lines sum to zero, so we're left with

$$\sum_{r=1,2}\left[\frac{p'\cdot\epsilon}{p'\cdot k}-\frac{p\cdot\epsilon}{p\cdot k}\right]^2=-\left(\frac{p'}{p'\cdot k}\right)^2-\left(\frac{p}{p\cdot k}\right)^2+2\frac{p'\cdot p}{(p'\cdot k)(p\cdot k)} \tag{8}$$

$$=-\left(\frac{p'}{p'\cdot k}-\frac{p}{p\cdot k}\right)^2 \tag{9}$$

so the differential cross section summed over polarizations is

$$\frac{d\sigma}{d\Omega}=-\left(\frac{d\sigma}{d\Omega}\right)_0\frac{\alpha}{(2\pi)^2}\int_{\text{IR}}\frac{d^3k}{\omega}\left(\frac{p'}{p'\cdot k}-\frac{p}{p\cdot k}\right)^2 \tag{10}$$