PARITY AND TIME REVERSAL AS LORENTZ TRANSFORMATIONS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 10, Exercise 10.1.

The operation of parity is one in which the spatial coordinates are reflected through the origin, so that a position vector $\mathbf{r}$ transforms to $-\mathbf{r}$. In matrix form, it is represented by

$$\mathsf{P}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix} \tag{1}$$

Time reversal, as its name implies, reverses the time coordinate so that $t\to-t$. Its matrix is

$$\mathsf{T}=\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{2}$$

The most general definition of a Lorentz transformation is one that leaves the metric tensor $g^{\mu\nu}$ invariant. That is, for a Lorentz transformation Λ, we must have

$$g^{\mu\nu}=\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}g^{\rho\sigma} \tag{3}$$

For parity and time-reversal, we have

$$\mathsf{P}^2=\mathsf{T}^2=\mathsf{I} \tag{4}$$

where I is the 4×4 identity matrix, so in these cases, they are obviously Lorentz transformations.

Both P and T are *improper* Lorentz transformations, meaning that they cannot be obtained from the identity matrix by a sequence of infinitesimal transformations. That is, we can't generate them by successively applying a transformation of the form

$$\Lambda^{\mu\nu} = g^{\mu\nu} + \omega^{\mu\nu} \tag{5}$$

where $\omega^{\mu\nu}$ is an infinitestimal quantity. Another way of saying this is that a proper Lorentz transformation has a determinant of $+1$ and an improper transformation has a determinant of -1.