PARITY TRANSFORMATION FOR VECTOR AND AXIAL VECTOR FIELDS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 10, Exercises 10.7-10.8.

By requiring the Dirac Lagrangian to be invariant under parity, L&P show that the parity matrix must satisfy the three conditions

$$\mathsf{P}^{\dagger}\mathsf{P} = 1 \tag{1}$$

$$\gamma_0\mathsf{P}^{\dagger}\gamma_0\gamma_i\mathsf{P} = -\gamma_i \tag{2}$$

$$\gamma_0\mathsf{P}^{\dagger}\gamma_0\mathsf{P} = 1 \tag{3}$$

The parity matrix that satisfies these three conditions is

$$\mathsf{P} = \eta_P\gamma_0 \tag{4}$$

where η_P is a phase factor, typically $\eta_P = \pm 1$.

L&P study the interaction term from quantum electrodynamics, which is

$$\mathscr{L}_{\text{int}} = -eQ\overline{\psi}\gamma_\mu\psi A^\mu \tag{5}$$

where e is the elementary charge, Q is a multiplier to give the charge on the fermion and A^μ is the electromagnetic field operator. They show that for this to be parity invariant, the field operators must transform as

$$A_P^0(x) = A^0(\tilde{x}) \tag{6}$$

$$\mathbf{A}_P(x) = -\mathbf{A}(\tilde{x}) \tag{7}$$

Vector fields that transform this way are called, simply, vector fields. If the vector field transformed with opposite parity, so that

$$A_P^0(x) = -A^0(\tilde{x}) \tag{8}$$

$$\mathbf{A}_P(x) = \mathbf{A}(\tilde{x}) \tag{9}$$

then they are called axial vector fields. We've seen that, for free fields, both choices leave Maxwell's equations invariant, so there is no way to distinguish between them unless we have an interaction term.

Now suppose we have an interaction term given by

$$\mathscr{L}_{\text{int}} = a\overline{\psi}\gamma^{\mu}\gamma_5\psi B_{\mu} \tag{10}$$

where a is a constant and B_{μ} is a spin-1 field (so it is either a vector or an axial vector). If we require the Lagrangian to be parity invariant, then

$$\mathcal{P}a\overline{\psi}\gamma^{\mu}\gamma_5\psi B_{\mu}\mathcal{P}^{-1} = \overline{\psi}(\tilde{x})\gamma_0\gamma^{\mu}\gamma_5\gamma_0\psi(\tilde{x})B_{\mu}(\tilde{x}) \tag{11}$$

$$= -\overline{\psi}(\tilde{x})\gamma_0\gamma_0\gamma^0\gamma_5\psi(\tilde{x})\eta_a B_0(\tilde{x}) + \overline{\psi}(\tilde{x})\gamma_0\gamma_0\gamma^i\gamma_5\psi(\tilde{x})\eta_b B_i(\tilde{x}) \tag{12}$$

$$= -\overline{\psi}(\tilde{x})\gamma^0\gamma_5\psi(\tilde{x})\eta_a B_0(\tilde{x}) + \overline{\psi}(\tilde{x})\gamma^i\gamma_5\psi(\tilde{x})\eta_b B_i(\tilde{x}) \tag{13}$$

Here, we've used the anticommutator properties of the gamma matrices, and I've inserted a couple of phase factors η_a and η_b which we must adjust to make the Lagrangian parity invariant. Comparing the last couple of lines with the original term 10, we see that for parity invariance we must have $\eta_a = -1$ and $\eta_b = +1$ which means that B_{μ} transforms like 9 and therefore is an axial vector field.

If we combine the QED interaction 5 with the interaction 10, we get a Lagrangian term like this:

$$\mathscr{L}_{\text{int}} = \overline{\psi}\gamma^{\mu}(a + b\gamma_5)\psi Z_{\mu} \tag{14}$$

where Z_{μ} is some spin-1 field and a and b are constants, we see that the term involving a behaves like the QED term 5 and the term involving b behaves like our earlier example 10. Thus the first term is parity invariant only if Z_{μ} is a vector field, and the second term is invariant only if Z_{μ} is an axial vector field, so we cannot make the total Lagrangian parity invariant.

PINGBACKS

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