CHARGE CONJUGATION IN QUANTUM ELECTRODYNAMICS

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Post date: 6 Jan 2019.

References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 10, Exercise 10.15.

As shown in L&P's equation 10.37, charge conjugation of a fermion field ψ is defined to give a new field ψ_C such that

$$\psi_C = \mathsf{C}\gamma_0^{\mathsf{T}}\psi^* \tag{1}$$

where C is the unitary charge conjugation matrix and

$$\psi^* \equiv \psi^{\dagger\mathsf{T}} \tag{2}$$

In section 10.3.2, L&P show that in interactions involving fermion fields, the fields always appear in a bilinear form such as

$$\overline{\psi}_1 F \psi_2 \tag{3}$$

where ψ_1 and ψ_2 need not be the same fermion field, and F is a constant 4×4 matrix. They then show that the applying charge conjugation to a term like this results in the term

$$\left(\overline{\psi}_1\right)_C F\left(\psi_2\right)_C = \overline{\psi}_2\left(\mathsf{C}^{-1}F\mathsf{C}\right)^{\mathsf{T}}\psi_1 \tag{4}$$

As an example, we can apply this to the interaction term in quantum electrodynamics, which is

$$\mathscr{L}_{\text{int}} = -eQ\overline{\psi}\gamma^\mu\psi A_\mu \tag{5}$$

where e is the elementary charge (the charge on a proton) and Q is an integer, so that eQ is the charge on the particle, and A_μ is the photon field. Using 4 we have for the charge conjugate

$$\left(\mathscr{L}_{\text{int}}\right)_C = -eQ\overline{\psi}\left(\mathsf{C}^{-1}\gamma^\mu\mathsf{C}\right)^{\mathsf{T}}\psi\left(A_\mu\right)_C \tag{6}$$

We can use the relation

$$\mathsf{C}^{-1}\gamma_\mu\mathsf{C} = -\gamma_\mu^{\mathsf{T}} \tag{7}$$

to get

$$\left(\mathsf{C}^{-1}\gamma^{\mu}\mathsf{C}\right)^{\mathsf{T}} = -\left(\gamma^{\mu\mathsf{T}}\right)^{\mathsf{T}} \tag{8}$$

$$= -\gamma^{\mu} \tag{9}$$

We therefore have

$$\left(\mathscr{L}_{\text{int}}\right)_C = eQ\overline{\psi}\gamma^{\mu}\psi\left(A_{\mu}\right)_C \tag{10}$$

In order for the Lagrangian to be invariant, we therefore must have

$$\left(A_{\mu}\right)_C = -A_{\mu} \tag{11}$$

which makes sense, because changing the sign of the charges will change the sign of all four components of the field A_{μ}.

PINGBACKS

Pingback: CP transformations in fermion interactions