CP TRANSFORMATIONS IN FERMION INTERACTIONS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 10, Exercise 10.18.

We can apply charge conjugation and parity together to a fermion field, as shown by L&P in section 10.5. The derivation goes like this.

$$\mathcal{CP}\psi(x)(\mathcal{CP})^{-1} = \mathcal{CP}\psi(x)\mathcal{P}^{-1}\mathcal{C}^{-1} \tag{1}$$

$$= \mathcal{C}\eta_P\gamma_0\psi(\tilde{x})\mathcal{C}^{-1} \tag{2}$$

$$= \eta_P\gamma_0\mathcal{C}\psi(\tilde{x})\mathcal{C}^{-1} \tag{3}$$

$$= \eta_P\eta_C\gamma_0\mathsf{C}\gamma_0^{\mathsf{T}}\psi^*(\tilde{x}) \tag{4}$$

where we used the parity transformation

$$\mathcal{CP}\psi(x)\mathcal{P}^{-1} = \mathsf{P}\psi(\tilde{x}) = \eta_P\gamma_0\psi(\tilde{x}) \tag{5}$$

to get the second line, and the charge conjugation matrix C defined by

$$\psi_C \equiv \mathcal{C}\psi(x)\mathcal{C}^{-1} = \eta_C\mathsf{C}\gamma_0^{\mathsf{T}}\psi^* \tag{6}$$

to get the last line. The quantities η_P and η_C are phases, each of which typically takes the values ±1.

Using the property

$$\mathsf{C}^{-1}\gamma_\mu\mathsf{C} = -\gamma_\mu^{\mathsf{T}} \tag{7}$$

we can write 4 as

$$\mathcal{CP}\psi(x)(\mathcal{CP})^{-1} = -\eta_{CP}\mathsf{C}\psi^*(\tilde{x}) \tag{8}$$

where

$$\eta_{CP} \equiv \eta_C\eta_P \tag{9}$$

and

$$\psi^* = \psi^{\dagger\mathsf{T}} \tag{10}$$

We can use 8 to work out the transformation of a typical fermion bilinear term $\overline{\psi}_1(x) F\psi_2(x)$, with F a constant matrix. We have

$$\mathcal{CP}\overline{\psi}_1(x) F\psi_2(x)(\mathcal{CP})^{-1} = \mathcal{CP}\overline{\psi}_1(x)(\mathcal{CP})^{-1} F\mathcal{CP}\psi_2(x)(\mathcal{CP})^{-1} \quad (11)$$

For the first term, using 8 and the unitary property $\mathsf{C}^\dagger = \mathsf{C}^{-1}$:

$$\mathcal{CP}\overline{\psi}_1(x)(\mathcal{CP})^{-1} = \mathcal{CP}\psi_1^\dagger(x)\gamma_0(\mathcal{CP})^{-1} \quad (12)$$
$$= -\eta_{CP}\psi_1^{*\dagger}(\tilde{x})\mathsf{C}^\dagger\gamma_0 \quad (13)$$
$$= -\eta_{CP}\psi_1^{\mathsf{T}}(\tilde{x})\mathsf{C}^{-1}\gamma_0 \quad (14)$$

Putting this back into 11 and using 8 again, we have (assuming η_{CP} is the same for both fields):

$$\mathcal{CP}\overline{\psi}_1(x) F\psi_2(x)(\mathcal{CP})^{-1} = (-\eta_{CP})^2\psi_1^{\mathsf{T}}(\tilde{x})\mathsf{C}^{-1}\gamma_0 F\mathsf{C}\psi^*(\tilde{x}) \quad (15)$$
$$= \psi_1^{\mathsf{T}}(\tilde{x})\mathsf{C}^{-1}\gamma_0 F\mathsf{C}\psi^*(\tilde{x}) \quad (16)$$

We can now apply the relation

$$\left(\overline{\psi}_1\right)_C F(\psi_2)_C = \overline{\psi}_2\left(\mathsf{C}^{-1}F\mathsf{C}\right)^{\mathsf{T}}\psi_1 \quad (17)$$

However, to do this, we need to identify the charge-conjugated terms $\left(\overline{\psi}_1\right)_C$ and $(\psi_2)_C$. First, we use 6 and 7 to show

$$\overline{\psi}_C = \psi_C^\dagger\gamma_0 \quad (18)$$
$$= \eta_C\psi^{\mathsf{T}}(\tilde{x})\gamma_0^{\mathsf{T}}\mathsf{C}^{-1}\gamma_0 \quad (19)$$
$$= -\eta_C\psi^{\mathsf{T}}(\tilde{x})\gamma_0^{\mathsf{T}}\gamma_0^{\mathsf{T}}\mathsf{C}^{-1} \quad (20)$$
$$= -\eta_C\psi^{\mathsf{T}}(\tilde{x})\mathsf{C}^{-1} \quad (21)$$

For ψ_C we have from 6 and 7

$$\psi_C = \eta_C\mathsf{C}\gamma_0^{\mathsf{T}}\psi^*(\tilde{x}) \quad (22)$$
$$= -\eta_C\gamma_0\mathsf{C}\psi^*(\tilde{x}) \quad (23)$$

so

$$\gamma_0\psi_C = -\eta_C\mathsf{C}\psi^*(\tilde{x}) \quad (24)$$

Substituting these two results back into 16 we have

$$\mathcal{CP}\overline{\psi}_1(x)F\psi_2(x)(\mathcal{CP})^{-1} = (-\eta_C)^2\left(\overline{\psi}_1\right)_C\gamma_0 F\gamma_0(\psi_2)_C \tag{25}$$

$$= \left(\overline{\psi}_1\right)_C\gamma_0 F\gamma_0(\psi_2)_C \tag{26}$$

We can now apply 17 to get (again using 7):

$$\left(\overline{\psi}_1\right)_C\gamma_0 F\gamma_0(\psi_2)_C = \overline{\psi}_2\left(\mathsf{C}^{-1}\gamma_0 F\gamma_0\mathsf{C}\right)^{\mathsf{T}}\psi_1 \tag{27}$$

$$= \overline{\psi}_2\left(\left(-\gamma_0^{\mathsf{T}}\mathsf{C}^{-1}\right)F\left(-\mathsf{C}\gamma_0^{\mathsf{T}}\right)\right)^{\mathsf{T}}\psi_1 \tag{28}$$

$$= \overline{\psi}_2\gamma_0\mathsf{C}^{\mathsf{T}}F^{\mathsf{T}}\left(\mathsf{C}^{-1}\right)^{\mathsf{T}}\gamma_0\psi_1 \tag{29}$$

We can now use the fact that C is antisymmetric so that $\mathsf{C}^{\mathsf{T}} = -\mathsf{C}$, to get

$$\left(\overline{\psi}_1\right)_C\gamma_0 F\gamma_0(\psi_2)_C = \overline{\psi}_2\gamma_0\mathsf{C}F^{\mathsf{T}}\mathsf{C}^{-1}\gamma_0\psi_1 \tag{30}$$

Thus we have

$$\boxed{\mathcal{CP}\overline{\psi}_1(x)F\psi_2(x)(\mathcal{CP})^{-1} = \overline{\psi}_2(\tilde{x})\gamma_0\mathsf{C}F^{\mathsf{T}}\mathsf{C}^{-1}\gamma_0\psi_1(\tilde{x})} \tag{31}$$

PINGBACKS

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