CP TRANSFORMATIONS OF INTERACTION MATRICES

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 10, Exercise 10.19.

We've seen that a general fermion bilinear term transforms under charge-parity CP as

$$\mathcal{C}\mathcal{P}\overline{\psi}_1(x)F\psi_2(x)(\mathcal{C}\mathcal{P})^{-1}=\overline{\psi}_2(\tilde{x})\gamma_0\mathsf{C}F^{\mathsf{T}}\mathsf{C}^{-1}\gamma_0\psi_1(\tilde{x}) \tag{1}$$

where F is a constant interaction matrix, typically composed of gamma matrices. If we define

$$F_{CP}\equiv\gamma_0\mathsf{C}F^{\mathsf{T}}\mathsf{C}^{-1}\gamma_0 \tag{2}$$

we can use the properties of the gamma matrices to work out F_{CP} for various values of F.

For $F=1$, we have

$$F_{CP}=\gamma_0\mathsf{C}F^{\mathsf{T}}\mathsf{C}^{-1}\gamma_0 \tag{3}$$

$$\gamma_0\mathsf{C}\mathsf{C}^{-1}\gamma_0 \tag{4}$$

$$=1 \tag{5}$$

For $F=\gamma_5$ we can use the relation

$$\mathsf{C}^{-1}\gamma_5\mathsf{C}=\gamma_5^{\mathsf{T}} \tag{6}$$

to get

$$F_{CP}=\gamma_0\mathsf{C}\gamma_5^{\mathsf{T}}\mathsf{C}^{-1}\gamma_0 \tag{7}$$

$$=\gamma_0\gamma_5\gamma_0 \tag{8}$$

$$=-\gamma_5\gamma_0\gamma_0 \tag{9}$$

$$=-\gamma_5 \tag{10}$$

For $F=\gamma_0$ we can use the relation

$$\mathsf{C}^{-1}\gamma_\mu\mathsf{C}=-\gamma_\mu^{\mathsf{T}} \tag{11}$$

to get

$$F_{CP} = \gamma_0 \mathsf{C} \gamma_0^{\mathsf{T}} \mathsf{C}^{-1} \gamma_0 \quad (12)$$
$$= -\gamma_0 \gamma_0 \gamma_0 \quad (13)$$
$$= -\gamma_0 \quad (14)$$

For $F = \gamma_i$ we again use 11 to get

$$F_{CP} = \gamma_0 \mathsf{C} \gamma_i^{\mathsf{T}} \mathsf{C}^{-1} \gamma_0 \quad (15)$$
$$= -\gamma_0 \gamma_i \gamma_0 \quad (16)$$
$$= \gamma_i \gamma_0 \gamma_0 \quad (17)$$
$$= \gamma_i \quad (18)$$

For $F = \gamma_0 \gamma_5$ we can use 10 and 14 to get

$$F_{CP} = \gamma_0 \mathsf{C} \left(\gamma_0 \gamma_5\right)^{\mathsf{T}} \mathsf{C}^{-1} \gamma_0 \quad (19)$$
$$= \gamma_0 \mathsf{C} \gamma_5^{\mathsf{T}} \mathsf{C}^{-1} \mathsf{C} \gamma_0^{\mathsf{T}} \mathsf{C}^{-1} \gamma_0 \quad (20)$$
$$= -\gamma_0 \gamma_5 \gamma_0 \gamma_0 \quad (21)$$
$$= -\gamma_0 \gamma_5 \quad (22)$$

Finally, for $F = \gamma_i \gamma_5$ we have

$$F_{CP} = \gamma_0 \mathsf{C} \left(\gamma_i \gamma_5\right)^{\mathsf{T}} \mathsf{C}^{-1} \gamma_0 \quad (23)$$
$$= \gamma_0 \mathsf{C} \gamma_5^{\mathsf{T}} \mathsf{C}^{-1} \mathsf{C} \gamma_i^{\mathsf{T}} \mathsf{C}^{-1} \gamma_0 \quad (24)$$
$$= -\gamma_0 \gamma_5 \gamma_i \gamma_0 \quad (25)$$
$$= -\gamma_0 \gamma_0 \gamma_5 \gamma_i \quad (26)$$
$$= -\gamma_5 \gamma_i \quad (27)$$
$$= \gamma_i \gamma_5 \quad (28)$$

To summarize, we have

F	1	γ_5	γ_0	γ_i	$\gamma_0\gamma_5$	$\gamma_i\gamma_5$
F_{CP}	1	$-\gamma_5$	$-\gamma_0$	γ_i	$-\gamma_0\gamma_5$	$\gamma_i\gamma_5$