ELECTRIC DIPOLE AND ANAPOLE MOMENTS

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References: Amitabha Lahiri & P. B. Pal, *A First Book of Quantum Field Theory*, Second Edition (Alpha Science International, 2004) - Chapter 11, Exercise 11.2.

We've seen that a general vertex including electromagnetic interactions is written as

$$\left\langle \mathbf{p}', s' \left| j_\mu (x) \right| \mathbf{p}, s \right\rangle = \frac{e^{-iq\cdot x}}{\sqrt{2E_p V}\sqrt{2E_{p'} V}} \overline{u}_{s'} \left(\mathbf{p}'\right) e\Gamma_\mu \left(p, p'\right) u_s (\mathbf{p}) \tag{1}$$

where the object Γ_μ is a 4×4 matrix with the general form

$$\Gamma_\mu = \gamma_\mu F_1 + \left(iF_2 + \tilde{F}_2\gamma_5\right)\sigma_{\mu\nu}q^\nu + \tilde{F}_3\left(q_\mu \not{q} - q^2\gamma_\mu\right)\gamma_5 \tag{2}$$

where

$$q \equiv p - p' \tag{3}$$

and the various F_i and $\tilde{F}_i$ are electromagnetic form factors, all of which are functions of q^2 because of Lorentz invariance.

In their section 11.2, L&P go through analyses to determine the physical interpretations of the various form factors. I won't reproduce the analysis here as for the most part it is explained clearly enough in the text. I'll just give a summary of the results.

Considering the charge form factor $F_1\left(q^2\right)$ for a general photon field A^μ, expanding $F_1\left(q^2\right)$ in a Taylor series about $q^2 = 0$ leads to the Dirac magnetic moment

$$\mu_D = \frac{eQ}{2m}\sigma = \frac{eQ}{m}\mathbf{S} \tag{4}$$

where $\mathbf{S} = \frac{1}{2}\sigma$ is the spin vector for the particle.

The form factor F_2 leads to the anomalous magnetic moment

$$\mu_A = -eF_2(0)\sigma = -2eF_2(0)\mathbf{S} \tag{5}$$

The electric dipole moment is given as

$$\mathbf{d}_E = e\tilde{F}_2(0)\,\sigma \tag{6}$$

and the anapole moment is $\tilde{F}_3(0)$ which appears in the total Lagrangian as the term

$$e\tilde{F}_3(0)\,\sigma\cdot\mathbf{j} \tag{7}$$

In exercise 11.2, L&P ask us to verify that the electric dipole and anapole moment interactions violate parity. I'm not sure how to about this, that is, whether we need to invoke the parity operator for fermions and do a bunch of calculations, or whether we can argue from physical principles.

To do the latter, we can argue as follows, although I'm not convinced it's actually correct.

Angular momentum in classical physics is defined as $\mathbf{L} = \mathbf{r}\times\mathbf{p}$ and since both $\mathbf{r}$ and $\mathbf{p}$ change sign under parity (inversion of the coordinates), $\mathbf{L}$ remains unchanged. The problem with trying to apply this argument to quantum theory is that the spin σ, although an angular momentum, isn't a classical quantity and isn't calculated from a cross product; rather, it's just an intrinsic property of a particle. It also doesn't give the correct result for $\mathbf{d}_E$ since it would seem to imply that $\mathbf{d}_E$ is even under parity.

Classically, an electric dipole is formed when two equal and opposite charges lie close to each other, with the dipole moment being a vector pointing in the direction from the negative to the positive charge. Inverting the coordinate system wouldn't change the absolute direction of the dipole moment in space, but in terms of coordinates it would invert, so classically, the dipole moment is odd under parity.

The anapole moment involves the current $\mathbf{j}$ which is also odd under parity, since the absolute direction in space of the current flow doesn't change, but the coordinate system used to describe it does, so $\mathbf{j}$ would become $-\mathbf{j}$. Thus it would seem that both the dipole and anapole moment interactions are odd under parity.

As I say, this is certainly not a satisfactory answer since it applies classical concepts to quantum objects. If anyone knows of a genuinely quantum explanation, please do leave a comment.