SINGLE-PARTICLE DENSITY MATRIX

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 4.2.

Post date: 4 Mar 2019.

We can define a one-particle density matrix by

$$\rho_1(\mathbf{x}-\mathbf{y}) \equiv \left\langle \psi^\dagger(\mathbf{x})\,\psi(\mathbf{y})\right\rangle \tag{1}$$

I'm not sure what the angle brackets mean in this definition. They don't appear to be part of a bra or ket. Often angle brackets indicate an average over space, but that doesn't seem to apply here, since the definition is in terms of locations $\mathbf{x}$ and $\mathbf{y}$.

In any case, the field $\psi(\mathbf{x})$ is here defined as

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\mathbf{p}} a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \tag{2}$$

with the adjoint

$$\psi^\dagger(\mathbf{x}) = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\mathbf{p}} a^\dagger_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{x}} \tag{3}$$

Substituting these into 1 we have

$$\rho_1(\mathbf{x}-\mathbf{y}) = \left\langle \frac{1}{\mathcal{V}}\sum_{\mathbf{p}}\sum_{\mathbf{q}} a^\dagger_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{x}} a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{y}} \right\rangle \tag{4}$$

$$= \frac{1}{\mathcal{V}}\sum_{\mathbf{p},\mathbf{q}} e^{-i(\mathbf{q}\cdot\mathbf{x}-\mathbf{p}\cdot\mathbf{y})}\left\langle a^\dagger_{\mathbf{q}} a_{\mathbf{p}}\right\rangle \tag{5}$$

Again, I'm not sure what the $\langle\rangle$ notation means. It doesn't seem to be an average over either position or momentum, and applies to operators only. In non-relativistic quantum mechanics, the density matrix is defined as

$$\rho \equiv \sum_i p_i \left|i\right\rangle \left\langle i\right| \tag{6}$$

where p_i is the probability of the system being in state i. The notation $\left\langle a^\dagger_{\mathbf{q}} a_{\mathbf{p}}\right\rangle$ might therefore indicate the probability of the incoming particle having momentum **p** and outgoing momentum **q** multiplied by the corresponding operators. Comments welcome.