LAGRANGIAN FOR A RELATIVISTIC FREE PARTICLE

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 5.4.

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Given the Lagrangian for a relativistic free particle as

$$L=-\frac{mc^2}{\gamma}=-\sqrt{1-\frac{v^2}{c^2}}mc^2 \tag{1}$$

we can calculate a few quantities in the non-relativistic limit $v \ll c$. In that case

$$\sqrt{1-\frac{v^2}{c^2}}\approx 1-\frac{v^2}{2c^2} \tag{2}$$

so we have

$$L\to -mc^2+\frac{1}{2}mv^2 \tag{3}$$

The momentum is defined as

$$p=\frac{\partial L}{\partial \dot{q}} \tag{4}$$

where in this case, $\dot{q}=v$, so we have

$$p\to mv \tag{5}$$

The Hamiltonian itself is

$$\begin{aligned} H &= p\dot{q}-L & (6)\\ &= mv^2+mc^2-\frac{1}{2}mv^2 & (7)\\ &= mc^2+\frac{1}{2}mv^2 & (8) \end{aligned}$$

which is the total energy (rest energy mc^2 + kinetic energy $\frac{1}{2}mv^2$).

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