MOMENTUM & ENERGY FOR A FREE PARTICLE IN AN ELECTROMAGNETIC FIELD

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 5.7.

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The Lagrangian for a free particle in an electromagnetic field is

$$L=-\frac{mc^2}{\gamma}+q\mathbf{A}(x)\cdot\mathbf{v}-qV(x) \tag{1}$$

In the non-relativistic limit, we've seen that for a totally free particle (no EM field), the Lagrangian becomes

$$L=-\frac{mc^2}{\gamma}=-\sqrt{1-\frac{v^2}{c^2}}mc^2\to-mc^2+\frac{1}{2}mv^2 \tag{2}$$

In this limit, the Lagrangian 1 thus becomes

$$L\to-mc^2+\frac{1}{2}mv^2+q\mathbf{A}(x)\cdot\mathbf{v}-qV(x) \tag{3}$$

The canonical momentum is

$$p_i=\frac{\partial L}{\partial v_i} \tag{4}$$

$$=mv_i+qA_i \tag{5}$$

The energy is given by the Hamiltonian

$$E=p_iv_i-L \tag{6}$$

$$=mv_iv_i+qA_iv_i+mc^2-\frac{1}{2}mv_iv_i-qA_iv_i+qV \tag{7}$$

$$=mc^2+\frac{1}{2}mv_iv_i+qV \tag{8}$$

We can eliminate the velocity in favour of the momentum by using 5:

$$v_i=\frac{1}{m}(p_i-qA_i) \tag{9}$$

so we get

$$E = mc^2 + \frac{1}{2m}(p_i - qA_i)(p_i - qA_i) + qV \tag{10}$$

In vector notation, this is

$$E = mc^2 + \frac{1}{2m}(\mathbf{p} - q\mathbf{A})^2 + qV \tag{11}$$