LAGRANGIAN EXAMPLE: INFINITE POLYNOMIAL IN THE SCALAR FIELD

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 7.1.

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An example Lagrangian is given by

I don't know if this Lagrangian describes an actual physical system.

$$\mathcal{L}=\frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi-\frac{1}{2}m^{2}\phi^{2}-\sum_{n=1}^{\infty}\lambda_{n}\phi^{2n+2} \tag{1}$$

Using the Euler-Lagrange equations to get the equation of motion we must find

$$\partial_{\mu}\frac{\partial\mathcal{L}}{\partial\left(\partial_{\mu}\phi\right)}-\frac{\partial\mathcal{L}}{\partial\phi}=0 \tag{2}$$

We have

$$-\frac{\partial\mathcal{L}}{\partial\phi}=m^{2}\phi+\sum_{n=1}^{\infty}\left(2n+2\right)\lambda_{n}\phi^{2n+1} \tag{3}$$

$$\partial_{\mu}\frac{\partial\mathcal{L}}{\partial\left(\partial_{\mu}\phi\right)}=\partial_{\mu}\partial^{\mu}\phi \tag{4}$$

$$=\partial^{2}\phi \tag{5}$$

Combining these two derivatives gives the equation of motion as

$$\left(\partial^{2}+m^{2}\right)\phi+\sum_{n=1}^{\infty}\left(2n+2\right)\lambda_{n}\phi^{2n+1}=0 \tag{6}$$