TIME EVOLUTION OF SPIN IN A MAGNETIC FIELD

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 8.4.

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The Heisenberg equation of motion for an operator Q is

$$\frac{dQ}{dt} = -\frac{i}{\hbar}[Q,H] \tag{1}$$

where H is the Hamiltonian.

Although we've looked at a spin-$\frac{1}{2}$ particle in a magnetic field before, here we'll do a simplified treatment. The Hamiltonian for the interaction of a particle with a constant magnetic field along the y direction can be written as

$$H = \omega S_y \tag{2}$$

where ω is a constant proportional to the magnetic field. The time evolution of the other two spin components can then be found from 1. We have

$$\frac{dS_z}{dt} = -\frac{i}{\hbar}[S_z,\omega S_y] \tag{3}$$
$$= -\frac{i\omega}{\hbar}[S_z,S_y] \tag{4}$$
$$= \frac{i\omega}{\hbar}[S_y,S_z] \tag{5}$$
$$= \frac{i\omega}{\hbar}(i\hbar S_x) \tag{6}$$
$$= -\omega S_x \tag{7}$$

where to get the fourth line, we've used the angular momentum commutator

$$[S_y,S_z] = i\hbar S_x \tag{8}$$

Similarly, for S_x we have

$$\frac{dS_x}{dt} = -\frac{i}{\hbar}[S_x, \omega S_y] \tag{9}$$

$$= -\frac{i\omega}{\hbar}[S_x, S_y] \tag{10}$$

$$= \omega S_z \tag{11}$$

Thus the components of spin perpendicular to the magnetic field change proportionally to each other. This is the precession of the spin about the magnetic field.