

## GENERATORS OF THE TRANSLATION OPERATOR

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 9.1.

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We've seen earlier that for an infinitesimal translation  $\varepsilon$  of the coordinate  $x$ , the translation operator is given by (in natural units so that  $\hbar = 1$ ):

$$U(\varepsilon) = I - i\varepsilon P \quad (1)$$

where  $P$  is the momentum operator. For a finite translation  $a$ , this operator becomes

$$U(a) = e^{-iaP} \quad (2)$$

or, for a 3-d translation  $\mathbf{a}$ , and using hats to indicate explicitly which items are operators

$$\hat{U}(\mathbf{a}) = e^{-i\mathbf{a}\cdot\hat{\mathbf{P}}} \quad (3)$$

From this last form, we can regain the generator as follows:

$$\left. \frac{\partial \hat{U}(\mathbf{a})}{\partial \mathbf{a}} \right|_{\mathbf{a}=0} = -i\hat{\mathbf{P}} e^{-i\mathbf{a}\cdot\hat{\mathbf{P}}} \Big|_{\mathbf{a}=0} \quad (4)$$

$$= -i\hat{\mathbf{P}} \quad (5)$$

Thus the generator is given by

$$\hat{\mathbf{P}} = -\frac{1}{i} \left. \frac{\partial \hat{U}(\mathbf{a})}{\partial \mathbf{a}} \right|_{\mathbf{a}=0} \quad (6)$$