**NONRELATIVISTIC COMPLEX SCALAR FIELD:
EULER-LAGRANGE EQUATIONS**

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 12.5.

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In their section 12.3, L&B write the Lagrangian for the nonrelativistic version of the complex scalar field for the free particle case ($V = 0$) as

$$\mathcal{L} = i\Psi^{\dagger}(x)\,\partial_0\Psi(x) - \frac{1}{2m}\nabla\Psi^{\dagger}(x)\cdot\nabla\Psi(x) \tag{1}$$

The Euler-Lagrange equations are

$$\frac{\partial\mathcal{L}}{\partial\Psi} = \partial_{\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\Psi)}\right) \tag{2}$$

We get

$$\frac{\partial\mathcal{L}}{\partial\Psi} = 0 \tag{3}$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_0\Psi)} = i\Psi^{\dagger}(x) \tag{4}$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_i\Psi)} = -\frac{1}{2m}\partial_i\Psi^{\dagger}(x) \tag{5}$$

For the conjugate field, we have

$$\frac{\partial\mathcal{L}}{\partial\Psi^{\dagger}} = i\partial_0\Psi(x) \tag{6}$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_0\Psi^{\dagger})} = 0 \tag{7}$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_i\Psi^{\dagger})} = -\frac{1}{2m}\partial_i\Psi(x) \tag{8}$$

Using the second form, we have from 2

$$i\partial_0\Psi(x) = -\frac{1}{2m}\nabla^2\Psi(x) \tag{9}$$

which is Schrödinger's equation for a free particle. This has the solution

$$\Psi(x) = Ae^{-ip\cdot x} \tag{10}$$

which leads to

$$i\partial_0\Psi(x) = Ap_0e^{-ip\cdot x} = E_{\mathbf{p}}\Psi(x) \tag{11}$$

$$-\frac{1}{2m}\nabla^2\Psi(x) = \frac{\mathbf{p}^2}{2m}\Psi(x) \tag{12}$$

so we have

$$E_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} \tag{13}$$

which is the nonrelativistic result.

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