NONRELATIVISTIC COMPLEX SCALAR FIELD: NOETHER CURRENT

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 12.6.

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For the nonrelativistic complex scalar field, we can find the Noether current and check that it makes sense in the nonrelativistic context. To find the current, we start with the Lagrangian in the form

$$\mathcal{L}=i\Psi^{\dagger}(x)\,\partial_{0}\Psi(x)-\frac{1}{2m}\nabla\Psi^{\dagger}(x)\cdot\nabla\Psi(x) \tag{1}$$

The momenta are

$$\Pi_{\Psi}^{0}=\frac{\partial\mathcal{L}}{\partial(\partial_{0}\Psi)}=i\Psi^{\dagger} \tag{2}$$

$$\Pi_{\Psi^{\dagger}}^{0}=\frac{\partial\mathcal{L}}{\partial(\partial_{0}\Psi^{\dagger})}=0 \tag{3}$$

$$\Pi_{\Psi}^{i}=\frac{\partial\mathcal{L}}{\partial(\partial_{i}\Psi)}=-\frac{1}{2m}\partial^{i}\Psi^{\dagger} \tag{4}$$

$$\Pi_{\Psi^{\dagger}}^{i}=\frac{\partial\mathcal{L}}{\partial(\partial_{i}\Psi^{\dagger})}=-\frac{1}{2m}\partial^{i}\Psi \tag{5}$$

Under the transformation

$$\Psi\to\Psi^{\prime}=e^{i\alpha}\Psi \tag{6}$$

we have

$$D\Psi=\left.\frac{\partial\Psi^{\prime}}{\partial\alpha}\right|_{\alpha=0}=i\Psi \tag{7}$$

$$D\Psi^{\dagger}=\left.\frac{\partial\Psi^{\dagger\prime}}{\partial\alpha}\right|_{\alpha=0}=-i\Psi^{\dagger} \tag{8}$$

The conserved current then has components

$$J_N^0 = \Pi_\Psi^0 D\Psi + \Pi_{\Psi^\dagger}^0 D\Psi^\dagger \tag{9}$$

$$= \left(i\Psi^\dagger\right)(i\Psi) + 0 \tag{10}$$

$$= -|\Psi|^2 \tag{11}$$

$$J_N^i = \Pi_\Psi^i D\Psi + \Pi_{\Psi^\dagger}^i D\Psi^\dagger \tag{12}$$

$$= -\frac{i}{2m}\left(\Psi\partial^i\Psi^\dagger - \Psi^\dagger\partial^i\Psi\right) \tag{13}$$

$$\mathbf{J}_N = -\frac{i}{2m}\left(\Psi\nabla\Psi^\dagger - \Psi^\dagger\nabla\Psi\right) \tag{14}$$

Thus J_N^0 represents the probability density $|\Psi|^2$ and $\mathbf{J}_N$ represents the probability current. In fact the Noether currents give the negatives of these two quantities, but if the negative is conserved, so is the positive quantity. We could have obtained the positive versions of both these quantities if we'd considered $e^{-i\alpha}\Psi$ in 6 instead.