SPIN OF THE PHOTON

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 14.2.

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In problem 14.2, L&B state that it can be shown using Noether's theorem that the operator S^z, whose eigenvalue is the z component of the photon's spin, obeys the commutation relation

$$\left[S^z, a_{\mathbf{q}\lambda}^\dagger\right] = i\epsilon_\lambda^{\mu=1*}(q)\, a_{\mathbf{q}\lambda=2}^\dagger - i\epsilon_\lambda^{\mu=2*}(q)\, a_{\mathbf{q}\lambda=1}^\dagger \tag{1}$$

where $\epsilon_\lambda^\mu(q)$ is the polarization vector where we're taking the two polarization vectors to be transvere:

$$\epsilon_{\lambda=1}^\mu = (0,1,0,0) \tag{2}$$
$$\epsilon_{\lambda=2}^\mu = (0,0,1,0) \tag{3}$$

The creation operator $a_{\mathbf{q}\lambda}^\dagger$ creates a photon of momentum **q** in polarization state λ.

We can define creation operators for left L and right R circular polarization states by

$$b_{\mathbf{q}R}^\dagger = -\frac{1}{\sqrt{2}}\left(a_{\mathbf{q}1}^\dagger + i a_{\mathbf{q}2}^\dagger\right) \tag{4}$$
$$b_{\mathbf{q}L}^\dagger = \frac{1}{\sqrt{2}}\left(a_{\mathbf{q}1}^\dagger - i a_{\mathbf{q}2}^\dagger\right) \tag{5}$$

Using 1 we can work out the commutators of S^z with these creation operators. We have

$$\left[S^z, b^\dagger_{\mathbf{q}R}\right] = -\frac{1}{\sqrt{2}}\left(\left[S^z, a^\dagger_{\mathbf{q}1}\right] + i\left[S^z, a^\dagger_{\mathbf{q}2}\right]\right) \tag{6}$$

$$= -\frac{1}{\sqrt{2}}\left(i\epsilon_1^{1*}a^\dagger_{\mathbf{q}2} - i\epsilon_1^{2*}a^\dagger_{\mathbf{q}1} + i\left(i\epsilon_2^{1*}a^\dagger_{\mathbf{q}2} - i\epsilon_2^{2*}a^\dagger_{\mathbf{q}1}\right)\right) \tag{7}$$

$$= -\frac{1}{\sqrt{2}}\left(a^\dagger_{\mathbf{q}1}\left(i\epsilon_1^{2*} + \epsilon_2^{2*}\right) + a^\dagger_{\mathbf{q}2}\left(i\epsilon_1^{1*} - \epsilon_2^{1*}\right)\right) \tag{8}$$

$$= -\frac{1}{\sqrt{2}}\left(a^\dagger_{\mathbf{q}1} + ia^\dagger_{\mathbf{q}2}\right) \tag{9}$$

$$= b^\dagger_{\mathbf{q}R} \tag{10}$$

where to get the fourth line, we inserted the components of the ϵ vectors using 2.

Similarly, we can get the other commutator:

$$\left[S^z, b^\dagger_{\mathbf{q}L}\right] = \frac{1}{\sqrt{2}}\left(a^\dagger_{\mathbf{q}1}\left(-i\epsilon_1^{2*} - \epsilon_2^{2*}\right) + a^\dagger_{\mathbf{q}2}\left(i\epsilon_1^{1*} + \epsilon_2^{1*}\right)\right) \tag{11}$$

$$= \frac{1}{\sqrt{2}}\left(-a^\dagger_{\mathbf{q}1} + ia^\dagger_{\mathbf{q}2}\right) \tag{12}$$

$$= -b^\dagger_{\mathbf{q}L} \tag{13}$$

Using these commutators, we can work out the effect of S^z on a state containing a single circularly polarized photon. Such a state with a right-polarized photon is given by $b^\dagger_{\mathbf{q}R}|0\rangle$ so we have, using 10

$$S^z b^\dagger_{\mathbf{q}R}|0\rangle = \left(b^\dagger_{\mathbf{q}R} + b^\dagger_{\mathbf{q}R}S^z\right)|0\rangle \tag{14}$$

$$= b^\dagger_{\mathbf{q}R}|0\rangle + 0 \tag{15}$$

$$= b^\dagger_{\mathbf{q}R}|0\rangle \tag{16}$$

where in the second line, we used the fact that the spin operator gives zero when acting on the vacuum state.

Similarly for $b^\dagger_{\mathbf{q}L}$ we have, using 13

$$S^z b^\dagger_{\mathbf{q}L}|0\rangle = \left(-b^\dagger_{\mathbf{q}L} + b^\dagger_{\mathbf{q}L}S^z\right)|0\rangle \tag{17}$$

$$= -b^\dagger_{\mathbf{q}L}|0\rangle + 0 \tag{18}$$

$$= -b^\dagger_{\mathbf{q}L}|0\rangle \tag{19}$$

Thus the z component of spin has eigenvalues ± 1.