SCALARS, PSEUDOSCALARS, VECTORS & PSEUDOVECTORS

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 15.2.

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The parity transformation reverses the signs of all three coordinate axes, and is equivalent to a reflection in a mirror followed by a rotation of π about the axis perpendicular to the mirror. Although rotation is a continuous operation, reflection is not, so a parity transformation is not a continuous transformation.

An ordinary vector (that is, one not obtained from a cross product) reverses under parity, so that $\mathbf{v}\to-\mathbf{v}$. The cross product of two ordinary vectors is therefore unchanged by parity; since both its constituent vectors change sign, the cross product is unchanged. An ordinary vector (one that *does* change sign under parity) is a *polar vector*; a cross product-type vector is a *pseudovector* or *axial vector*.

An ordinary scalar (such as temperature or density) remains unchanged by parity. However, a scalar formed by a triple product of 3 polar vectors, as in $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$, *does* change sign under parity, and is known as a *pseudoscalar*. A dot product of a polar with an axial vector also gives a pseudoscalar.

Some examples:

- Magnetic flux $\mathbf{B}$ is a pseudovector, since it is formed from the curl of the magnetic vector potential: $\nabla\times\mathbf{A}$. The curl operator ∇ behaves like a polar vector, since it contains the derivatives with respect to the three coordinates, and these all change sign under parity.
- Angular momentum $\mathbf{L}$ is a pseudovector, since it is the cross product of two polar vectors: $\mathbf{L}=\mathbf{r}\times\mathbf{p}$.
- Charge q is a scalar, as it is an intrinsic property of a particle, and not the result of a vector triple product.
- The scalar product of a vector and a pseudovector is a pseudoscalar, since the vector changes sign but the pseudovector does not. Thus the scalar product does change sign.
- The scalar product of two polar vectors is an ordinary scalar, since both vectors change sign and their product therefore does not.

- The scalar product of two pseudovectors is also a scalar, since neither pseudovector changes sign.