# GREEN FUNCTION FOR INFINITE SQUARE WELL

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 16.1.

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The Green function for the Schrödinger equation has the form

$$G^{+}(x,t_x,y,t_y)=\theta(t_x-t_y)\sum_n \phi_n(x)\,\phi_n^*(y)\,e^{-iE_n(t_x-t_y)} \tag{1}$$

where $\phi_n$ is an eigenfunction of the hamiltonian $H$ with energy $E_n$. By using various transformations, the Green function can be written in terms of other variables. In L&B's Example 16.7 they show that in the momentum-time domain, we have

$$G_0^{+}(p,t_x,q,t_y)=\theta(t_x-t_y)\,\delta(p-q)\,e^{-iE_p(t_x-t_y)} \tag{2}$$

where $q$ and $p$ are the momenta in the initial and final states. The delta function conserves momentum so that we always have $p=q$. As a result, sometimes the Green function is written in a simplified form by omitting the delta function:

$$G_0^{+}(p,t_x,t_y)=\theta(t_x-t_y)\,e^{-iE_p(t_x-t_y)} \tag{3}$$

In the case of a particle in the infinite square well of width $a$, the energies are (with $\hbar=1$):

$$E_n=\frac{n^2\pi^2}{2ma^2} \tag{4}$$

The Green function in this case is therefore

$$G_0^{+}(n,t_x,t_y)=\theta(t_x-t_y)\,e^{-i\frac{n^2\pi^2}{2ma^2}(t_x-t_y)} \tag{5}$$

where we've written the momentum in the argument of $G_0^{+}$ using the quantum number $n$.

We can also write the Green function in the momentum-energy domain by doing a Fourier transform to eliminate the time, as is done in L&B's Example 16.8. The result is

$$G_0^+(p,E) = \frac{i}{E - E_p + i\epsilon} \tag{6}$$

where the infinitesimal quantity $\epsilon$ was introduced to ensure converge of the integral in the Fourier transform. For the square well, we have

$$G_0^+(p,E) = \frac{i}{E - \frac{n^2\pi^2}{2ma^2} + i\epsilon} \tag{7}$$

L&B use the symbol $\omega$ which I assume is the energy of the particle, so with that notation we have

$$G_0^+(n,\omega) = \frac{i}{\omega - \frac{n^2\pi^2}{2ma^2} + i\epsilon} \tag{8}$$