COMMUTATOR OF CONSERVED CHARGE WITH FIELD

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Chapter 10.

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In their discussion of Noether's theorem, L&B consider a symmetry transformation in which the transformation is parameterized by a quantity called λ. For an infinitesimal transformation, the field ϕ changes by an amount given by the first term in a Taylor expansion:

$$\delta\phi=\left.\frac{\partial\phi}{\partial\lambda}\right|_{\lambda=0}\delta\lambda \tag{1}$$

The derivative term is given the shorthand notation:

$$D\phi\equiv\left.\frac{\partial\phi}{\partial\lambda}\right|_{\lambda=0} \tag{2}$$

so that

$$\delta\phi=D\phi\,\delta\lambda \tag{3}$$

A derivation of Noether's theorem follows in the book, with the result that the locally conserved Noether current is given by

$$J_N^{\mu}(x)=\Pi^{\mu}(x)\,D\phi(x)-W^{\mu}(x) \tag{4}$$

where, for a given Lagrangian density $\mathcal{L}$

$$\Pi^{\mu}(x)=\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\phi)} \tag{5}$$

is the momentum density and $W^{\mu}(x)$ is some function whose four-divergence can be added to the Lagrangian without affecting the symmetry transformation (see L&B section 10.2 for details).

The conserved Noether charge is given by

$$Q_N=\int d^3x\;J_N^0(x) \tag{6}$$

In the common case where $W^{\mu}=0$, this becomes

$$Q_N = \int d^3x\, \Pi^0(x) D\phi(x) \tag{7}$$

Using the usual commutation relation between a field and its conjugate momentum

$$\left[\phi(x), \Pi^0(y)\right] = i\delta^{(3)}(\mathbf{x}-\mathbf{y}) \tag{8}$$

we have

$$[Q_N, \phi(y)] = \int d^3x\, \left[\Pi^0(x), \phi(y)\right] D\phi(x) \tag{9}$$

$$= -\int d^3x\, \left[\phi(y), \Pi^0(x)\right] D\phi(x) \tag{10}$$

$$= -i\int d^3x\, \delta^{(3)}(\mathbf{x}-\mathbf{y}) D\phi(x) \tag{11}$$

$$= -iD\phi(y) \tag{12}$$

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