TENSORS IN MAPLE - DEFINITION AND INDEX ACTIONS

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I've used Maple for many calculations on physicspages, so I thought it would be worthwhile to write a few posts on how to use it. I'll start with an introduction to tensors.

Although Maple is designed to do mathematical operations, there is a dedicated Physics package which contains facilities for doing many mathematical operations related to physics. This package was first introduced in Maple 2017. I'm using Maple 2021 so it's possible that some of these features will not be available in earlier editions.

If you want to export the expression to Latex, you'll need the `maple.sty` Latex style file, which can be found in the 'etc' directory of the Maple installation. Despite my attempts to get the Miktex program to install this file globally, I couldn't get it to work, but if you just include maple.sty in the same directory as your Latex document and put the command `\usepackage{maple}` in the Latex preamble for the document, the style file is included and all is well.

All Maple worksheets that use the Physics package must begin with

```
with(Physics):
```

The colon at the end stops Maple from printing out a lengthy response to this command. After this line, we can use any function in the Physics package.

To begin, we'll consider the realm of special relativity. The most basic tensor used in special relativity is the metric tensor $g_{\mu\nu}$, so we'll see how to set this up in Maple. The signature we've been using for relativity is to have the t component negative and the three spatial components positive. This isn't the default in Maple, so we need to change that.

The Setup() command is used to set most of the properties for physics calculations. To set the coordinate system so that the coordinates are in the order $[t,x,y,z]$ and the signature is $[-,+,+,+]$ we use

```
Setup(coordinates = {X = [t, x, y, z]}, signature = `-+++`);
```

The delimiters used in the signature are back-quotes, not apostrophes.

The metric tensor is denoted by `g_[]` and we can check that we have the correct metric by typing this as a command. The result is

$$g_{\mu,\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Maple inserts a comma between the μ and ν in $g_{\mu,\nu}$ which can be a bit confusing, as the comma is often used as the notation for a derivative with respect to x^{ν}, so you need to be a bit careful here.

With the metric defined, we can now define an ordinary tensor. A rank-1 tensor (a vector or one-form) can be defined by

```
Define(A[mu])
```

Maple responds with

$$\{A_{\mu}, \gamma_{\mu}, \sigma_{\mu}, \partial_{\mu}, g_{\mu,\nu}, \epsilon_{\alpha,\beta,\mu,\nu}, X_{\mu}\} \tag{2}$$

This is a list of all tensors currently defined in the session. Maple includes some commonly-found tensors in physics as standard. They are the γ matrices used in the Dirac equation in quantum mechanics, the σ matrices (Pauli matrices), also from quantum mechanics, the differential operator ∂_{μ} indicating a derivative with respect to x^{μ}, the metric tensor $g_{\mu,\nu}$ in 1 and the anti-symmetric Levi-Civita tensor $\epsilon_{\alpha,\beta,\mu,\nu}$. The X_{μ} is just the set of 4 coordinates defined earlier.

The definition `A[mu]` defines a tensor with a covariant (lower) index. To get a contravariant (upper) index, prefix the index with ~, so we have

```
Define(C[~alpha])
```

which gives C^{α}.

We can define tensors with higher rank by listing the indices. So we can have

```
Define(B[mu,nu])
```

which gives us $B_{\mu\nu}$ and so on.

Multiplying tensors can be done by just writing out the product. For example, we can write

```
g_[mu, nu]A[alpha]B[mu, nu]
```

Maple's response is to write

$$g_{\mu\nu} A_{\alpha} B^{\mu\nu} \tag{3}$$

Notice that, although we wrote the product with all indices in the lower position, Maple recognizes repeated indices (μ and ν here) and raises one of each pair so that the summation convention can be applied to these indices.

We can use the Simplify() command to recognize that the metric tensor in 3 lowers an index from $B^{\mu\nu}$, so we get

`Simplify(%)`
where the % argument means to apply Simplify to the previous result, which gives the result

$$A_\alpha B_\nu{}^\nu \tag{4}$$

We can then use the `SumOverRepeatedIndices(%)` command to sum over the ν index, with the result

$$A_\alpha \left(B_1{}^1 + B_2{}^2 + B_3{}^3 + B_4{}^4\right) \tag{5}$$

PINGBACKS

Pingback: Tensors in Maple - definition from tensor equations