COMPLEX ARITHMETIC

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Post date: 2 November 2024.

Here are a few examples of complex arithmetic. Any standard arithmetic expression involving complex numbers can be reduced to the form $a+bi$.

$$(8+i)-(5+i)=3 \tag{1}$$

$$\frac{2}{i}=\frac{2}{i}\frac{(-i)}{(-i)}=-2i \tag{2}$$

$$(-1+i)^2=1-2i+i^2=-2i \tag{3}$$

$$\frac{2-i}{1/3}=6-3i \tag{4}$$

$$i(\pi-4i)=4+\pi i \tag{5}$$

$$\frac{8i-1}{i}=-i(8i-1)=8+i \tag{6}$$

$$\frac{-1+5i}{2+3i}=\frac{-1+5i}{2+3i}\times\frac{2-3i}{2-3i}=\frac{1}{13}(13+13i)=1+i \tag{7}$$

$$\frac{3}{i}+\frac{i}{3}=-3i+\frac{1}{3}i=-\frac{8}{3}i \tag{8}$$

$$\begin{aligned}\frac{(8+2i)-(1-i)}{(2+i)^2} &= \frac{(8+2i)-(1-i)}{(2+i)^2}\times\frac{(2-i)^2}{(2-i)^2} && (9)\\ &= \frac{1}{25}\left[(8+2i)-(1-i)\right](2-i)^2 && (10)\\ &= \frac{1}{25}(7+3i)(3-4i) && (11)\\ &= \frac{1}{25}(33-19i) && (12)\end{aligned}$$

$$\frac{2+3i}{1+2i}-\frac{8+i}{6-i} = \frac{2+3i}{1+2i}\frac{1-2i}{1-2i}-\frac{8+i}{6-i}\frac{6+i}{6+i} \tag{13}$$
$$= \frac{1}{5}(2+3i)(1-2i)-\frac{1}{37}(8+i)(6+i) \tag{14}$$
$$= \frac{8-i}{5}-\frac{47+14i}{37} \tag{15}$$
$$= \frac{61}{185}-\frac{107}{185}i \tag{16}$$

$$\left[\frac{2+i}{6i-(1-2i)}\right]^2 = \left(\frac{2+i}{-1+8i}\right)^2 \tag{17}$$
$$= \left(\frac{2+i}{-1+8i}\right)^2\left(\frac{-1-8i}{-1-8i}\right)^2 \tag{18}$$
$$= \frac{1}{65^2}(2+i)^2(-1-8i)^2 \tag{19}$$
$$= \frac{1}{4225}(3+4i)(-63+16i) \tag{20}$$
$$= \frac{1}{4225}(-253-204i) \tag{21}$$

At this point, it's convenient to note a property of the powers of i.

$$i^2 = -1 \tag{22}$$
$$i^3 = -i \tag{23}$$
$$i^4 = 1 \tag{24}$$

We can generalize this to

$$i^{4k} = \left(i^4\right)^k = 1 \tag{25}$$
$$i^{4k+1} = \left(i^{4k}\right)i = i \tag{26}$$
$$i^{4k+2} = \left(i^{4k}\right)i^2 = -1 \tag{27}$$
$$i^{4k+3} = \left(i^{4k}\right)i^3 = -i \tag{28}$$

As examples we have

$$i^7 = i^4 i^3 = -i \tag{29}$$

$$i^{62} = i^{4\times 15+2} = -1 \tag{30}$$

$$i^{-202} = \frac{1}{i^{4\times 50+2}} = \frac{1}{-1} = -1 \tag{31}$$

$$i^{-4321} = \frac{1}{i^{4\times 1080+1}} = \frac{1}{i} = -i \tag{32}$$

$$i^3 (1+i)^2 = -i\,(2i) = 2 \tag{33}$$