COMPLEX PLANE

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Complex numbers can be plotted in the xy plane by taking the real part as x and the imaginary part as y. Complex numbers can then be used to represent various geometric figures.

Example 1. $\Im z=-2$. This defines the line $y=-2$, a horizontal line passing through the y axis at -2.

Example 2. $|z-1+i|=3$. This defines the points that are at a distance 3 from the point $(1,-1)$, which is a circle of radius 3 centred at $(1,-1)$. We can also see this by doing the algebra. We square both sides of the equation to get

$$(x-1)^2+(y+1)^2=9 \tag{1}$$

which is the desired circle.

Example 3. $|2z-i|=4$. We have

$$(2x)^2+(2y-1)^2=16 \tag{2}$$

$$x^2+\left(y-\frac{1}{2}\right)^2=4 \tag{3}$$

which is a circle centred at $\left(0,\frac{1}{2}\right)$ with radius 2.

Example 4. $|z-1|=|z+i|$. Squaring both sides and cancelling terms:

$$(x-1)^2+y^2=x^2+(y+1)^2 \tag{4}$$

$$-2x=2y \tag{5}$$

$$y=-x \tag{6}$$

Thus this represents the diagonal line through the origin sloping downwards.

Example 5. $|z|=\Re z+2$. We have

$$x^2+y^2=(x+2)^2 \tag{7}$$
$$y^2=4x+4 \tag{8}$$
$$x=\frac{y^2}{4}-1 \tag{9}$$

This is a parabola opening to the right, with vertex at $(-1,0)$.

Example 6. $|z-1|+|z+1|=7$. Trying to work out this by expanding the terms is very tedious. However, we can note that the two modulus terms represent the distances from the points $(1,0)$ and $(-1,0)$. That is, the equation defines the curve where the sum of the distances from two fixed points is a constant (7), which is the definition of an ellipse with foci at $(1,0)$ and $(-1,0)$. Using Maple, we can grind through the algebra by expanding the modulus terms and squaring to get the equation of the ellipse, which is

$$\frac{4}{49}x^2+\frac{4}{45}y^2=1 \tag{10}$$

From the standard form of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we see that the semi-major axis is $a=7/2=3.5$ and the semi-minor axis is $b=3\sqrt{5}/2\approx 3.354$. This gives an eccentricity of

$$e=\frac{\sqrt{a^2-b^2}}{a}=\frac{4}{49}\approx 0.0816 \tag{11}$$

Thus the ellipse is very nearly a circle.

Example 7. $\Re z\ge 4$. This is the half-plane $x\ge 4$, that is, all points to the right of, and including, the vertical line $x=4$.

Example 8. $|z-i|<2$. This is all points inside the circle with centre $(0,1)$ and radius $\sqrt{2}$. As the relation is $<$, the circle itself is excluded.

Example 9. $|z|>6$. This is all points outside the circle with centre at the origin and radius $\sqrt{6}$.

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