CONTINUUM SETS

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A set of points in the complex plane is closed if it contains all its boundary points. An open set is connected if there is a polygonal path connecting any two points in the set. A closed set is defined to be connected if it cannot be written as the union of two nonempty disjoint (that is, non-intersecting) closed sets. A closed connected set is called a *continuum*.

continuum

Example 1. $\{z : |z-3| = 4\}$. This is the circle of radius 4 with centre at $(3,0)$. This cannot be written as the union of two disjoint closed subsets, so it is a continuum. Note that it's important that we consider only *closed* disjoint subsets. The circle *can* be written as the union of two disjoint subsets, where one subset is closed and the other is open. For example, we could define the closed subset to be the semicircle with angles in the range $[-\pi, 0]$ (the lower half of the circle). We can then define the open subset to be the upper half of the circle with angles in the range $(0, \pi)$, which excludes the endpoints of the semicircle.

Example 2. $\{z : |z| = 1\} \cup \{z : |z| = 3\}$. This set contains the two circles with centres at the origin and radii of 1 and 3 respectively. In this case, the two subsets are disjoint, since the two circles do not intersect, so this is not a continuum.

Example 3. $\{1, -1, i\}$. This set is merely three disjoint points, so it is not a continuum.

Example 4. $\{z : |z-1| \geq 2\}$. This defines all points outside of, and including, the circle of radius 2 with centre at $(1,0)$. This set cannot be written as the union of two disjoint closed subsets, so it is a continuum. Again, it's important to consider only disjoint closed subsets, as in Example 1.