COMPTON SCATTERING

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Compton scattering occurs when a photon scatters off another particle such as an electron at rest. As a result of energy transfer from the photon to the other particle, the photon's frequency decreases so that it in effect suffers a red shift. The size of the change in frequency depends on the scattering angle θ and the mass m of the other particle, and can be derived from conservation of energy and momentum.

We will analyze the situation in the frame in which the other particle is at rest before the collision. In that frame, the initial momentum of the particle is zero, so its four-momentum is

$$\vec{p}_e = (E,0,0,0) \tag{1}$$

Since the particle is at rest, its energy is just

$$E = m \tag{2}$$

The four-momentum of the photon is, assuming that its incident direction is along the x axis:

$$\begin{aligned} \vec{p}_\gamma &= (h\nu, h\nu, 0, 0) && (3)\\ &= (h\nu, \mathbf{p}) && (4) \end{aligned}$$

where we've written the last three components as a three-vector $\mathbf{p}$.

Recall that the momentum four-vector for a photon is a null vector, and the energy of a photon from quantum mechanics is $E = h\nu$, so the momentum of a photon travelling in the x direction must be numerically equal to its energy.

After the collision, the four-momentum of the photon is (using primed variables):

$$\vec{p}_\gamma^{\prime} = (h\nu', \mathbf{p}') \tag{5}$$

The particle is now moving, so will have a non-zero momentum vector

$$\vec{p}_e^{\prime} = (E', \mathbf{p}_e^{\prime}) \tag{6}$$

Using the relativistic formula for the relation between energy and momentum for a particle with rest mass m we get

$$E' = \sqrt{(p'_e)^2 + m^2} \tag{7}$$

From conservation of energy we get

$$\begin{aligned} E + h\nu &= E' + h\nu' && (8)\\ m + h\nu &= \sqrt{(p'_e)^2 + m^2} + h\nu' && (9) \end{aligned}$$

From conservation of momentum, we have

$$\begin{aligned} \mathbf{p}'_e + \mathbf{p}' &= \mathbf{p} && (10)\\ (p'_e)^2 &= (\mathbf{p} - \mathbf{p}')\cdot(\mathbf{p} - \mathbf{p}') && (11)\\ &= p^2 + (p')^2 - 2\mathbf{p}\cdot\mathbf{p}' && (12)\\ &= h^2(\nu^2 + (\nu')^2 - 2\nu\nu'\cos\theta) && (13) \end{aligned}$$

where θ is the angle between the incident and scattered directions for the photon.

Substituting this into the conservation of energy equation we get

$$(p'_e)^2 + m^2 = \left(m + h\nu - h\nu'\right)^2 \tag{14}$$

$$h^2(\nu^2 + (\nu')^2 - 2\nu\nu'\cos\theta) + m^2 = m^2 + h^2\left(\nu^2 + (\nu')^2\right) + 2h(m\nu - m\nu' - h\nu\nu') \tag{15}$$

$$-h\nu\nu'\cos\theta = m\nu - m\nu' - h\nu\nu' \tag{16}$$

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m}(1 - \cos\theta) \tag{17}$$

Note that the change in (inverse) frequency depends on a term that is inversely proportional to the mass of the particle. If the 'particle' is a macroscopic object such as a mirror, then m will be so large that this term is negligible, and the scattered (reflected) wavelength is essentially the same as the incident wavelength. It is for this reason that we don't observe reflections in mirrors to be grossly red-shifted. In this case, the momentum transmitted to the mirror is

$$p'_e = h\nu\sqrt{2(1 - \cos\theta)} \tag{18}$$

The formula is more normally given in terms of the angle of incidence ϕ of the photon to the mirror, which is (assuming specular reflection) $\phi = \theta/2$. Using the trig identity

$$\cos 2\phi = 1 - 2\sin^2\phi \tag{19}$$

we get

$$p_e' = 2h\nu\sin\phi \tag{20}$$

That is, the momentum transferred is twice the perpendicular component of the incident photon's momentum, which is required since that component of the photon's momentum is reversed in order to reflect it.

If the photon were absorbed, then the transfer would be $h\nu\sin\phi$.

PINGBACKS

Pingback: Compton effect; Compton wavelength