DOPPLER EFFECT

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The world line of a photon is a null line, so any vector tangent to a photon's world line is a null vector. This means in particular that a four-velocity cannot be defined for a photon, since all four-velocities have to satsify the condition

$$\vec{U}\cdot\vec{U}=-1 \tag{1}$$

Another consequence of this is that, if we require a photon's momentum to be parallel to its world line, then the momentum must also be a null vector. Since the four-momentum is defined as the vector $\vec{p}$ with p^0 equal to the energy and the other three components equal to the three-momentum, this means that for photons

$$E^2=\mathbf{p}\cdot\mathbf{p} \tag{2}$$

In particular, if the photon is moving in the x direction, then $p^1=E$ and $p^2=p^3=0$.

From quantum mechanics, we know (well, it's one of the postulates of quantum mechanics anyway) that

$$E=h\nu \tag{3}$$

where ν is the frequency of the photon and h is Planck's constant. We can combine this with the Lorentz transformation of the photon's four-momentum to get a formula for the Doppler shift.

The Doppler effect occurs because the observer is moving relative to a light source. If light is being emitted by a source such as a star, then the light will have a particular frequency (or in general, mixture of frequencies, but we'll concentrate on monochromatic light), which can be measured as the number of peaks in the wave that pass a fixed point in one second. If the observer moves towards the light source, then in that second, he will pass a greater number of peaks in the wave, and thus the frequency of the light appears higher, or *blue-shifted*, since for visible light, the colour appears shifted towards the blue end of the spectrum. Similarly, if the observer moves away from the light source, the frequency appears lower and the light is *red-shifted*.

Note that this effect does not violate the postulate of the constancy of the speed of light, which is fundamental to relativity. The light itself still moves at the same speed relative to the moving observers; what changes is the frequency, and hence the energy, of the light that is observed.

If the photon is moving at angle θ relative to the x axis, then assuming it is moving in the $x-y$ plane, its momentum is

$$\begin{aligned}\vec{p} &= (E, E\cos\theta, E\sin\theta, 0) &(4)\\ &= (h\nu, h\nu\cos\theta, h\nu\sin\theta, 0) &(5)\end{aligned}$$

Since $E = h\nu$, we use the Lorentz transformation for an observer moving at speed v along the x axis to get for $p^{\bar{0}} = h\bar{\nu}$ (be careful not to confuse the symbol ν (Greek lowercase nu) with v):

$$\begin{aligned}h\bar{\nu} &= h\nu(\gamma - \gamma v\cos\theta) &(6)\\ &= \frac{h\nu}{\sqrt{1-v^2}}(1 - v\cos\theta) &(7)\\ \frac{\bar{\nu}}{\nu} &= \frac{1 - v\cos\theta}{\sqrt{1-v^2}} &(8)\end{aligned}$$

In the special case where the photon is moving along the x axis, $\theta = 0$ and the formula becomes

$$\frac{\bar{\nu}}{\nu} = \sqrt{\frac{1-v}{1+v}} \tag{9}$$

If $v > 0$, this formula gives a red-shift, since $\bar{\nu} < \nu$. If $v < 0$, the direction of motion of the observer relative to the light is reversed and we get a blue-shift. Note that it is impossible for the Doppler shift to reduce a photon's frequency to zero, since this would require $v = 1$, and relativity forbids anything except massless particles (photons, mainly) from moving at the speed of light.

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