LORENTZ TRANSFORMATIONS FOR ENERGY AND MOMENTUM

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Post date: 28 Aug 2018.

We can work out the Lorentz transformations for relativistic energy and momentum. Suppose an object of rest mass m in frame S' is moving along the x' axis at a speed u'. Then its energy and momentum are

$$E' = \gamma_{S'} m \tag{1}$$

$$= \frac{m}{\sqrt{1-u'^2}} \tag{2}$$

$$p' = \gamma_{S'} m u' \tag{3}$$

$$= \frac{mu'}{\sqrt{1-u'^2}} \tag{4}$$

Now consider frame S moving at speed v along the x (or x'- the two axes coincide) axis relative to S'. In this frame, the object is moving at a different speed u and has energy and momentum

$$E = \gamma_S m \tag{5}$$

$$= \frac{m}{\sqrt{1-u^2}} \tag{6}$$

$$p = \gamma_S m u \tag{7}$$

$$= \frac{mu}{\sqrt{1-u^2}} \tag{8}$$

The two speeds u and u' are related by the velocity addition formula

$$u = \frac{u'+v}{1+vu'} \tag{9}$$

Therefore the energy in frame S is given by

$$E = \frac{m}{\sqrt{1-\left(\frac{u'+v}{1+vu'}\right)^2}} \tag{10}$$

Doing a bit of algebra we get

$$E = \frac{m(1+u'v)}{\sqrt{(1-u'^2)(1-v^2)}} \tag{11}$$

$$= \gamma_v \gamma_{S'} m \left(1+u'v\right) \tag{12}$$

$$= \gamma_v \left(E' + vp'\right) \tag{13}$$

For momentum, we have

$$p = \frac{m}{\sqrt{1-\left(\frac{u'+v}{1+vu'}\right)^2}} \frac{u'+v}{1+vu'} \tag{14}$$

$$= \gamma_v \gamma_{S'} m \left(u'+v\right) \tag{15}$$

$$= \gamma_v \left(p' + vE'\right) \tag{16}$$

Although we derived these formulas for a particle with non-zero rest mass, they also hold for zero rest mass particles such as photons. In this case, we have, in frame S', $E' = p'$ so

$$E = \gamma_v E' (1+v) \tag{17}$$

$$= E' \sqrt{\frac{1+v}{1-v}} \tag{18}$$

Since the energy of a photon is related to its frequency by $E = h\nu$ where h is Planck's constant, this is the Doppler effect formula.

PINGBACKS

Pingback: Electron & muon-neutrino scattering