FOUR-MOMENTUM OF PHOTONS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 3; Problem 3.4.

The four-momentum is defined for a particle with a rest mass m as

$$\mathbf{p}=\gamma m\left[1,v_{x},v_{y},v_{z}\right] \tag{1}$$

where $\gamma=1/\sqrt{1-v^{2}}$.

For particles such as photons that have no rest mass, this formula obviously doesn't work. However, the first component of $\mathbf{p}$ is taken as the particle's energy, so if we make that identification for photons, we get

$$\mathbf{p}=E\left[1,v_{x},v_{y},v_{z}\right] \tag{2}$$

For any massless particle, $v=\sqrt{v_{x}^{2}+v_{y}^{2}+v_{z}^{2}}=1$, so $\mathbf{p}\cdot\mathbf{p}=E^{2}\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}-1\right)=0$, making $\mathbf{p}$ a null vector.

As an example, suppose we have a positive pion π^{+} (rest mass 140 MeV) at rest that decays into an antimuon μ^{+} (rest mass 106 MeV) and a neutrino. Avoiding any controversy over whether or not the neutrino has mass, we'll just assume it's massless so its momentum is (assuming it travels along the $+x$ axis):

$$\mathbf{p}_{\nu}=E_{\nu}\left[1,1,0,0\right] \tag{3}$$

For the muon we have

$$\mathbf{p}_{\mu}=E_{\mu}\left[1,v,0,0\right] \tag{4}$$

where $E_{\mu}=106\gamma\,\text{MeV}$.

From conservation of momentum we get from the energy component

$$\begin{aligned} E_{\nu}+E_{\mu} &= E_{\pi} && (5)\\ E_{\nu}+106\gamma &= 140 && (6) \end{aligned}$$

From the second component we get

$$E_\nu + vE_\mu = 0 \tag{7}$$

$$E_\nu + 106v\gamma = 0 \tag{8}$$

Subtracting these two equations we get

$$106\gamma(1-v) = 140 \tag{9}$$

$$\frac{1-v}{\sqrt{1-v^2}} = 1.32 \tag{10}$$

$$1-2v+v^2 = 1.744-1.744v^2 \tag{11}$$

This quadratic has only one acceptable solution (there is also $v=1$ which is spurious since it makes γ infinite), which is $v=-0.271$. This gives $\gamma=1.0389$, from which we get the energies:

$$E_\nu = 140-106\gamma \tag{12}$$

$$= 29.88\text{ MeV} \tag{13}$$

$$E_\mu = 110.12\text{ MeV} \tag{14}$$

PINGBACKS

Pingback: Four-momentum conservation in electron-positron annihilation

Pingback: Doppler effect and four-momentum

Pingback: Schwarzschild metric: four-momentum of a photon

Pingback: Stress-energy tensor for a photon gas

Pingback: De Broglie waves