

## FOUR-MOMENTUM OF PHOTONS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 3; Problem 3.4.

The four-momentum is defined for a particle with a rest mass  $m$  as

$$(0.1) \quad \mathbf{p} = \gamma m [1, v_x, v_y, v_z]$$

where  $\gamma = 1/\sqrt{1-v^2}$ .

For particles such as photons that have no rest mass, this formula obviously doesn't work. However, the first component of  $\mathbf{p}$  is taken as the particle's energy, so if we make that identification for photons, we get

$$(0.2) \quad \mathbf{p} = E [1, v_x, v_y, v_z]$$

For any massless particle,  $v = \sqrt{v_x^2 + v_y^2 + v_z^2} = 1$ , so  $\mathbf{p} \cdot \mathbf{p} = E^2 (v_x^2 + v_y^2 + v_z^2 - 1) = 0$ , making  $\mathbf{p}$  a null vector.

As an example, suppose we have a positive pion  $\pi^+$  (rest mass 140 MeV) at rest that decays into an antimuon  $\mu^+$  (rest mass 106 MeV) and a neutrino. Avoiding any controversy over whether or not the neutrino has mass, we'll just assume it's massless so its momentum is (assuming it travels along the  $+x$  axis):

$$(0.3) \quad \mathbf{p}_\nu = E_\nu [1, 1, 0, 0]$$

For the muon we have

$$(0.4) \quad \mathbf{p}_\mu = E_\mu [1, v, 0, 0]$$

where  $E_\mu = 106\gamma$  MeV.

From conservation of momentum we get from the energy component

$$(0.5) \quad E_\nu + E_\mu = E_\pi$$

$$(0.6) \quad E_\nu + 106\gamma = 140$$

From the second component we get

$$(0.7) \quad E_\nu + \nu E_\mu = 0$$

$$(0.8) \quad E_\nu + 106\nu\gamma = 0$$

Subtracting these two equations we get

$$(0.9) \quad 106\gamma(1 - \nu) = 140$$

$$(0.10) \quad \frac{1 - \nu}{\sqrt{1 - \nu^2}} = 1.32$$

$$(0.11) \quad 1 - 2\nu + \nu^2 = 1.744 - 1.744\nu^2$$

This quadratic has only one acceptable solution (there is also  $\nu = 1$  which is spurious since it makes  $\gamma$  infinite), which is  $\nu = -0.271$ . This gives  $\gamma = 1.0389$ , from which we get the energies:

$$(0.12) \quad E_\nu = 140 - 106\gamma$$

$$(0.13) \quad = 29.88 \text{ MeV}$$

$$(0.14) \quad E_\mu = 110.12 \text{ MeV}$$

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