FOUR-MOMENTUM OF PHOTONS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 3; Problem 3.4.

The four-momentum is defined for a particle with a rest mass m as

$$\mathbf{p}=\gamma m\left[1,v_x,v_y,v_z\right] \tag{1}$$

where $\gamma=1/\sqrt{1-v^2}$.

For particles such as photons that have no rest mass, this formula obviously doesn't work. However, the first component of $\mathbf{p}$ is taken as the particle's energy, so if we make that identification for photons, we get

$$\mathbf{p}=E\left[1,v_x,v_y,v_z\right] \tag{2}$$

For any massless particle, $v=\sqrt{v_x^2+v_y^2+v_z^2}=1$, so $\mathbf{p}\cdot\mathbf{p}=E^2\left(v_x^2+v_y^2+v_z^2-1\right)=0$, making $\mathbf{p}$ a null vector.

As an example, suppose we have a positive pion π^+ (rest mass 140 MeV) at rest that decays into an antimuon μ^+ (rest mass 106 MeV) and a neutrino. Avoiding any controversy over whether or not the neutrino has mass, we'll just assume it's massless so its momentum is (assuming it travels along the $+x$ axis):

$$\mathbf{p}_\nu=E_\nu\left[1,1,0,0\right] \tag{3}$$

For the muon we have

$$\mathbf{p}_\mu=E_\mu\left[1,v,0,0\right] \tag{4}$$

where $E_\mu=106\gamma\,\mathrm{MeV}$.

From conservation of momentum we get from the energy component

$$\begin{aligned} E_\nu+E_\mu &= E_\pi && (5)\\ E_\nu+106\gamma &= 140 && (6)\end{aligned}$$

From the second component we get

$$E_{\nu}+vE_{\mu} = 0 \quad (7)$$
$$E_{\nu}+106v\gamma = 0 \quad (8)$$

Subtracting these two equations we get

$$106\gamma(1-v) = 140 \quad (9)$$
$$\frac{1-v}{\sqrt{1-v^2}} = 1.32 \quad (10)$$
$$1-2v+v^2 = 1.744-1.744v^2 \quad (11)$$

This quadratic has only one acceptable solution (there is also $v=1$ which is spurious since it makes γ infinite), which is $v=-0.271$. This gives $\gamma=1.0389$, from which we get the energies:

$$E_{\nu} = 140-106\gamma \quad (12)$$
$$= 29.88\,\text{MeV} \quad (13)$$
$$E_{\mu} = 110.12\,\text{MeV} \quad (14)$$

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