FOUR-MOMENTUM OF PHOTONS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 3; Problem 3.4.

The four-momentum is defined for a particle with a rest mass m as

$$\mathbf{p}=\gamma m\left[1,v_{x},v_{y},v_{z}\right] \tag{1}$$

where $\gamma=1/\sqrt{1-v^{2}}$.

For particles such as photons that have no rest mass, this formula obviously doesn't work. However, the first component of $\mathbf{p}$ is taken as the particle's energy, so if we make that identification for photons, we get

$$\mathbf{p}=E\left[1,v_{x},v_{y},v_{z}\right] \tag{2}$$

For any massless particle, $v=\sqrt{v_{x}^{2}+v_{y}^{2}+v_{z}^{2}}=1$, so $\mathbf{p}\cdot\mathbf{p}=E^{2}\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}-1\right)=0$, making $\mathbf{p}$ a null vector.

As an example, suppose we have a positive pion π^{+} (rest mass 140 MeV) at rest that decays into an antimuon μ^{+} (rest mass 106 MeV) and a neutrino. Avoiding any controversy over whether or not the neutrino has mass, we'll just assume it's massless so its momentum is (assuming it travels along the $+x$ axis):

$$\mathbf{p}_{\nu}=E_{v}\left[1,1,0,0\right] \tag{3}$$

For the muon we have

$$\mathbf{p}_{\mu}=E_{\mu}\left[1,v,0,0\right] \tag{4}$$

where $E_{\mu}=106\gamma$ MeV.

From conservation of momentum we get from the energy component

$$\begin{aligned}E_{\nu}+E_{\mu} &= E_{\pi} &(5)\\ E_{\nu}+106\gamma &= 140 &(6)\end{aligned}$$

From the second component we get

$$
\begin{aligned}
E_\nu + vE_\mu &= 0 && (7)\\
E_\nu + 106v\gamma &= 0 && (8)
\end{aligned}
$$

Subtracting these two equations we get

$$
\begin{aligned}
106\gamma(1-v) &= 140 && (9)\\
\frac{1-v}{\sqrt{1-v^2}} &= 1.32 && (10)\\
1-2v+v^2 &= 1.744-1.744v^2 && (11)
\end{aligned}
$$

This quadratic has only one acceptable solution (there is also $v=1$ which is spurious since it makes γ infinite), which is $v=-0.271$. This gives $\gamma=1.0389$, from which we get the energies:

$$
\begin{aligned}
E_\nu &= 140-106\gamma && (12)\\
&= 29.88\text{ MeV} && (13)\\
E_\mu &= 110.12\text{ MeV} && (14)
\end{aligned}
$$

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