FOUR-MOMENTUM CONSERVATION IN ELECTRON-POSITRON ANNIHILATION

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 3; Problem 3.5.

When an electron and positron (each with mass m) collide they annihilate each other and produce two photons. Suppose these two particles are each travelling with speed v towards each other along the x axis in an inertial frame. Before the collision, the total four-momentum is

$$\begin{aligned}\mathbf{p}_1 &= \gamma m[1,v,0,0]+\gamma m[1,-v,0,0] &(1)\\ &= \gamma m[2,0,0,0] &(2)\end{aligned}$$

where $\gamma=1/\sqrt{1-v^2}$.

After the collision, suppose the two photons travel in opposite directions along the x axis, and have the same energy E. Then the momentum after the collision is

$$\begin{aligned}\mathbf{p}_2 &= E[1,1,0,0]+E[1,-1,0,0] &(3)\\ &= E[2,0,0,0] &(4)\end{aligned}$$

Momentum is therefore conserved in the lab frame if

$$\begin{aligned}E &= \gamma m &(5)\\ &= \frac{m}{\sqrt{1-v^2}} &(6)\end{aligned}$$

Now suppose we look at the problem in the electron's frame. Before the collision, the situation is the same as the one we analyzed earlier, so the momentum before the collision is

$$\mathbf{p}_1'=2m\gamma^2[1,-v,0,0] \tag{7}$$

Since the electron's frame is moving at a speed v relative to the lab frame, we can use the Lorentz transformations on the photon momenta in the lab frame to find their values in the electron's frame. The transformations for the t and x components are

$$p'^t = \gamma p^t - v\gamma p^x \quad (8)$$
$$p'^x = -v\gamma p^t + \gamma p^x \quad (9)$$

Applying this to $\mathbf{p}_2$ we get

$$\mathbf{p}'_2 = \gamma E[1-v, 1-v, 0, 0] + \gamma E[1+v, -1-v, 0, 0] \quad (10)$$
$$= 2\gamma E[1, -v, 0, 0] \quad (11)$$

Since $E = \gamma m$, we see that $\mathbf{p}'_1 = \mathbf{p}'_2$ so four-momentum is conserved in the electron's frame as well.

Note that we could have just applied the Lorentz transformation to the final form of $\mathbf{p}_2$; we didn't need to work out the transformations on each photon separately. However it's interesting to see how the two photons transform. The energy of the photon travelling to the right is reduced by a factor of $\gamma(1-v) = \sqrt{(1-v)/(1+v)}$, while that of the photon moving to the left is increased by a factor of $\gamma(1+v) = \sqrt{(1+v)/(1-v)}$. Since the speeds of the photons remain unchanged (the speed of light), this change in energy is reflected in a change of their wavelengths. This is of course the Doppler effect.

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