ELECTRON-POSITRON COLLISION MUST PRODUCE AT LEAST 2 PHOTONS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 3; Problem 3.8.

When an electron and positron collide, they can decay into a pair of photons as we've seen. However, 2 photons is the minimum number of photons that can be produced. To see why a single photon is not possible, re-examine the solution we had earlier. In that post, we started with the electron and positron moving towards each other, one with velocity v and the other with velocity $-v$. We then transformed to the frame in which the electron is at rest. We can just as easily reverse the process, and start with the electron's frame. Let's say that in the electron's frame, the positron is moving with speed $-v'$ towards it. Then the total momentum before the collision is

$$\mathbf{p}' = [m,0] + \gamma' \left[m, -v'\right] \tag{0.1}$$

where $\gamma' = 1/\sqrt{1-(v')^2}$.

We can now transform to a frame in which the two particles are moving towards each other with the same speed v. That is, we want the four-velocities to be

$$\mathbf{u}_e = [\gamma m, v] \tag{0.2}$$

$$\mathbf{u}_p = [\gamma m, -v] \tag{0.3}$$

From our earlier analysis, we see that this works if

$$v' = \frac{2v}{1+v^2} \tag{0.4}$$

$$v = \frac{1-\sqrt{1-(v')^2}}{v'} \tag{0.5}$$

Thus it's always possible to find v such that the particles are moving towards each other with the same speed, and in that frame, the total momentum is

$$\mathbf{p} = [2\gamma m, 0] \tag{0.6}$$

Since the x component of a photon's momentum is always ± 1, there is no way this momentum can be conserved after the collision if only a single photon is emitted. However, it's always possible to produce 2 photons moving in opposite directions, since then the x component of momentum can add up to zero, as required.