FOUR-MOMENTUM CONSERVATION: ELECTRON-PHOTON COLLISION

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 3; Problem 3.10.

Here's another example of using the invariance of the scalar product under Lorentz transformations. This time we have a photon colliding with a stationary electron (in the lab frame) and producing an extra electron-positron pair (so we have 2 electrons and a positron after the collision; the photon gets absorbed so it disappears). What is the minimum energy of the incoming photon (in the lab frame) to achieve this?

First, we can look at the problem in the centre of mass (COM) frame. There, the electron travels at some speed v (the photon, of course, travels at $v=1$) and after the collision all 3 particles are at rest. From conservation of momentum we must have

$$\left(\mathbf{p}_e+\mathbf{p}_\gamma\right)^2=-\left(3m\right)^2 \tag{1}$$

where $\mathbf{p}_e$ is the momentum of the electron and $\mathbf{p}_\gamma$ is the momentum of the photon before the collision, and m is the mass of an electron or positron.

Multiplying this out, we get

$$\begin{aligned}
p_e^2+p_\gamma^2+2\mathbf{p}_e\cdot\mathbf{p}_\gamma &= -m^2-0+2\mathbf{p}_e\cdot\mathbf{p}_\gamma && (2)\\
&= -9m^2 && (3)\\
\mathbf{p}_e\cdot\mathbf{p}_\gamma &= -4m^2 && (4)
\end{aligned}$$

Since the scalar product on the LHS is invariant, it must also be true in the lab frame. There, $\mathbf{p}_e^{\prime}=[m,0]$ since the electron is at rest, and $\mathbf{p}_\gamma^{\prime}=[E,-E]$. Therefore

$$\begin{aligned}
\mathbf{p}_e^{\prime}\cdot\mathbf{p}_\gamma^{\prime} &= -mE && (5)\\
&= \mathbf{p}_e\cdot\mathbf{p}_\gamma=-4m^2 && (6)\\
E &= 4m && (7)
\end{aligned}$$

This gives us the minimum energy of the photon in the lab frame.

We can work out the wavelength of this photon by using $E = h/\lambda$ where Planck's constant has the value $h = 1240 \times 10^{-9}$ eV m and the electron mass is $m = 0.51 \times 10^6$ eV. We get

$$\lambda = \frac{h}{E} \tag{8}$$

$$= \frac{1240 \times 10^{-9}}{4 \times 0.51 \times 10^6} \tag{9}$$

$$= 6.08 \times 10^{-13}\text{m} \tag{10}$$

This is well into the gamma ray region of the electromagnetic spectrum.