FOUR-ACCELERATION

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 4; Problems 4.11.

The four-velocity $\mathbf{u}$ has the property that, for any object with a non-zero rest mass, $\mathbf{u} \cdot \mathbf{u} = -1$ (For a photon, four-velocity cannot be defined.). If we define the four-acceleration as

$$\mathbf{a} \equiv \frac{d\mathbf{u}}{d\tau} \tag{1}$$

then if we take the derivative of the square of the four-velocity, we get

$$\begin{aligned} \frac{d(\mathbf{u} \cdot \mathbf{u})}{d\tau} &= 0 && (2) \\ 2\mathbf{u} \cdot \frac{d\mathbf{u}}{d\tau} &= 0 && (3) \\ \mathbf{u} \cdot \mathbf{a} &= 0 && (4) \end{aligned}$$

Thus for any object with a non-zero rest mass, the four-velocity is orthogonal to the four-acceleration.