METRIC TENSOR: SEMI-LOG COORDINATES

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 5; Problem 5.4.

As an example of a different coordinate system, we can define the semi-log system by introducing coordinates p and q defined as:

$$p = x \tag{1}$$
$$q = e^{by} \tag{2}$$

where b is a constant. If x and y have units of length, then b must have units of length^{-1}. Note that this means that p and q have different units, with p having the units of length and q being dimensionless.

The curves of constant p are just the same as the curves of constant x, that is, vertical lines. The curves of constant q are defined by $e^{by} = k$ or $y = \frac{\ln k}{b}$. These are horizontal lines, although the spacing for equal steps in k will translate into the variable spacing seen on semi-log plots.

If an object has an acceleration $\mathbf{a} = a^i$ in the rectangular system, then we can find its acceleration in the semi-log system in the usual way.

$$a^p = \frac{\partial p}{\partial x}a^x + \frac{\partial p}{\partial y}a^y \tag{3}$$
$$= a^x \tag{4}$$
$$a^q = \frac{\partial q}{\partial x}a^x + \frac{\partial q}{\partial y}a^y \tag{5}$$
$$= be^{by}a^y \tag{6}$$

The units of a^p are still those of acceleration, but the units of a^q are length$^{-1}\cdot$acceleration.

We can work out the metric of the semi-log system from the rectangular metric by direct calculation:

$$g'_{ij} = g_{kl}\frac{\partial x^k}{\partial x'^i}\frac{\partial x^l}{\partial x'^j} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{(bq)^2} \end{bmatrix} \tag{7}$$

The length squared of $\mathbf{a}$ is an invariant, since

$$
\begin{aligned}
a^2 &= g'_{ij}a'^i a'^j && (8)\\
&= (a^x)^2+\left(\frac{1}{bq}\right)^2\left(be^{by}a^y\right)^2 && (9)\\
&= (a^x)^2+(a^y)^2 && (10)
\end{aligned}
$$

The basis vectors in the semi-log system have lengths obtainable from the the metric:

$$
\begin{aligned}
\mathbf{e}_p\cdot\mathbf{e}_p &= g_{pp} = 1 && (11)\\
\left|\mathbf{e}_p\right| &= 1 && (12)\\
\mathbf{e}_q\cdot\mathbf{e}_q &= g_{qq} = \frac{1}{(bq)^2} && (13)\\
\left|\mathbf{e}_q\right| &= \frac{1}{bq} && (14)
\end{aligned}
$$

Incidentally, the question part (e) as written in Moore's book doesn't make sense; he asks for the length of the basis vector ∂x. If we want to use partial derivatives as basis vectors, we need to define a curve along which to take the derivative. Since Moore doesn't even mention the use of partial derivatives as a basis in the text, I can only assume this is a typo.

PINGBACKS

Pingback: Metric tensor: spherical coordinates
Pingback: Covariant derivative in semi-log coordinates