

## METRIC TENSOR: SPHERICAL COORDINATES

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 5; Problem 5.6.

The non-rectangular coordinate systems (semi-log and sinusoidal) we've looked at so far have all been flat, so it's time to look at one in curved space. We'll use the surface of a sphere, but rather than the usual spherical coordinates we'll use a slight variation. We keep the azimuthal angle  $\phi$  but use as the second coordinate the quantity  $r$  which is the distance along the surface of the sphere measured from the north pole. If the radius of the sphere is  $R$ , then in terms of normal spherical coordinates,  $r = R\theta$ .

Curves of constant  $\phi$  are the usual lines of longitude, while curves of constant  $r$  are lines of latitude. The tangents to the two curves at a given point are always perpendicular, so the metric  $g_{ij}$  will be diagonal. To find the diagonal components, consider an infinitesimal displacement  $ds$ . We have

$$ds = dr\mathbf{e}_r + d\phi\mathbf{e}_\phi \quad (1)$$

and our job is to find the two basis vectors.

The displacement along  $\mathbf{e}_r$  is just  $dr = R d\theta$ , so  $\mathbf{e}_r$  is a unit vector. A displacement along  $\mathbf{e}_\phi$  depends on the radius of the constant  $r$  curve. In spherical coordinates, this is  $R \sin \theta$ , so in our new coordinate system we get the displacement as  $R \sin \theta d\phi = R \sin \frac{r}{R} d\phi$ . Therefore the magnitude of  $\mathbf{e}_\phi$  is  $R \sin \frac{r}{R}$ . The metric tensor is thus

$$g_{ij} = \begin{bmatrix} 1 & 0 \\ 0 & (R \sin \frac{r}{R})^2 \end{bmatrix} \quad (2)$$

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