METRIC TENSOR: SPHERICAL COORDINATES

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 5; Problem 5.6.

The non-rectangular coordinate systems (semi-log and sinusoidal) we've looked at so far have all been flat, so it's time to look at one in curved space. We'll use the surface of a sphere, but rather than the usual spherical coordinates we'll use a slight variation. We keep the azimuthal angle ϕ but use as the second coordinate the quantity r which is the distance along the surface of the sphere measured from the north pole. If the radius of the sphere is R, then in terms of normal spherical coordinates, $r = R\theta$.

Curves of constant ϕ are the usual lines of longitude, while curves of constant r are lines of latitude. The tangents to the two curves at a given point are always perpendicular, so the metric g_{ij} will be diagonal. To find the diagonal components, consider an infinitesimal displacement $d\mathbf{s}$. We have

$$d\mathbf{s} = dr\mathbf{e}_r + d\phi\mathbf{e}_\phi \tag{1}$$

and our job is to find the two basis vectors.

The displacement along $\mathbf{e}_r$ is just $dr = Rd\theta$, so $\mathbf{e}_r$ is a unit vector. A displacement along $\mathbf{e}_\phi$ depends on the radius of the constant r curve. In spherical coordinates, this is $R\sin\theta$, so in our new coordinate system we get the displacement as $R\sin\theta d\phi = R\sin\frac{r}{R}d\phi$. Therefore the magnitude of $\mathbf{e}_\phi$ is $R\sin\frac{r}{R}$. The metric tensor is thus

$$g_{ij} = \begin{bmatrix} 1 & 0 \\ 0 & \left(R\sin\frac{r}{R}\right)^2 \end{bmatrix} \tag{2}$$

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