METRIC TENSOR: PARABOLIC COORDINATES

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 5; Problem 5.7.

Another example of a non-rectangular 2-d curved coordinate system. This time we have a parabolic bowl with equation

$$z = br^2 \tag{0.1}$$

where $r^2 = x^2 + y^2$ and b is a constant. We use the two coordinates r (as defined here) and the azimuthal angle ϕ.

Using the same technique as with the sphere, we consider infinitesimal displacements along the constant curves. The curve of constant ϕ is a parabola, while the curve of constant r is a circle at height $z = br^2$. The tangents to the two curves at a given point are always perpendicular, so the metric g_{ij} will be diagonal. To find the diagonal components, consider an infinitesimal displacement $d\mathbf{s}$. We have

$$d\mathbf{s} = dr\mathbf{e}_r + d\phi\mathbf{e}_\phi \tag{0.2}$$

and our job is to find the lengths of the two basis vectors.

Consider first a displacement along $\mathbf{e}_r$. As r increases, we move a distance up the side of the parabolic bowl. This displacement consists of a horizontal increment of size dr and a vertical increment of size $dz = 2brdr$. By Pythagoras, the total displacement is $ds = \sqrt{1+(2br)^2}dr$. The length of $\mathbf{e}_r$ is therefore $\sqrt{1+(2br)^2}$.

A displacement along $\mathbf{e}_\phi$ is a displacement around a circle of constant r, so here $ds = rd\phi$, giving the length of $\mathbf{e}_\phi$ as r. The metric is then

$$g_{ij} = \begin{bmatrix} 1+(2br)^2 & 0 \\ 0 & r^2 \end{bmatrix} \tag{0.3}$$

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