

TENSOR TRACE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 6; Problem 6.3.

The *trace* of a rank-2 tensor is given by the contraction F^i_j . In matrix terminology, it is the sum of the diagonal elements. If we start with a contravariant tensor F^{ij} , then we can calculate the trace as follows:

$$(1) \quad F^i_j = g_{jk} F^{ik}$$
$$(2) \quad F^i_i = g_{ik} F^{ik}$$

That is, we first lower the second index, then contract the top and bottom indices.

Since the trace contains no free index, it should be a scalar, which means it should be invariant. We can prove this by doing the transformation.

$$(3) \quad g'_{ik} F'^{ik} = \frac{\partial x^l}{\partial x'^i} \frac{\partial x^m}{\partial x'^k} g_{lm} \frac{\partial x'^i}{\partial x^a} \frac{\partial x'^k}{\partial x^b} F^{ab}$$
$$(4) \quad = \left(\frac{\partial x^l}{\partial x'^i} \frac{\partial x'^i}{\partial x^a} \right) \left(\frac{\partial x^m}{\partial x'^k} \frac{\partial x'^k}{\partial x^b} \right) g_{lm} F^{ab}$$
$$(5) \quad = \delta^l_a \delta^m_b g_{lm} F^{ab}$$
$$(6) \quad = g_{ab} F^{ab}$$

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