TENSOR TRACE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 6; Problem 6.3.

The *trace* of a rank-2 tensor is given by the contraction $F^i{}_i$. In matrix terminology, it is the sum of the diagonal elements. If we start with a contravariant tensor F^{ij}, then we can calculate the trace as follows:

$$\begin{aligned} F^i{}_j &= g_{jk}F^{ik} && (1) \\ F^i{}_i &= g_{ik}F^{ik} && (2) \end{aligned}$$

That is, we first lower the second index, then contract the top and bottom indices.

Since the trace contains no free index, it should be a scalar, which means it should be invariant. We can prove this by doing the transformation.

$$\begin{aligned} g'_{ik}F'^{ik} &= \frac{\partial x^l}{\partial x'^i}\frac{\partial x^m}{\partial x'^k}g_{lm}\frac{\partial x'^i}{\partial x^a}\frac{\partial x'^k}{\partial x^b}F^{ab} && (3) \\ &= \left(\frac{\partial x^l}{\partial x'^i}\frac{\partial x'^i}{\partial x^a}\right)\left(\frac{\partial x^m}{\partial x'^k}\frac{\partial x'^k}{\partial x^b}\right)g_{lm}F^{ab} && (4) \\ &= \delta^l{}_a\delta^m_b g_{lm}F^{ab} && (5) \\ &= g_{ab}F^{ab} && (6) \end{aligned}$$

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