TENSOR INDICES: NEWTON'S LAW

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 6; Problem 6.8.

We've seen that Newton's law for a particle with mass m and charge q moving in an electromagnetic field given by the field tensor F^{ij} is

$$\frac{dp^i}{d\tau} = m\frac{du^i}{d\tau} \tag{1}$$

$$= qF^{ij}\eta_{ja}u^a \tag{2}$$

$$= qF^{ij}u_j \tag{3}$$

Moore offers 5 other options for this equation. The options $m\frac{du^i}{dt} = qF^{ij}$, $m\frac{du^i}{d\tau} = qF^{ij}$, $m\frac{du^i}{dt} = qF^{ij}u^j$ and $m\frac{du^i}{d\tau} = qF^{ij}u^j$ are all invalid since the tensor indices don't balance on each side of the equation. Remember that an implied sum occurs only if one repeated index is upper and the other is lower.

The option $m\frac{du^i}{dt} = qF^{ij}u_j$ is invalid, since the time t in the derivative is not an invariant between inertial frames. We must use the proper time τ.