

ELECTROMAGNETIC FIELD TENSOR: A COUPLE OF MAXWELL'S EQUATIONS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 6; Problem 6.10, 7.2.

The electromagnetic field tensor F^{ij} is

$$(0.1) \quad F^{ij} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{bmatrix}$$

The various electromagnetic laws can be expressed by the equation

$$(0.2) \quad \partial_i F_{jk} + \partial_k F_{ij} + \partial_j F_{ki} = 0$$

The lowered version F_{ij} of F^{ij} in the flat metric of special relativity is

$$(0.3) \quad F_{ij} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

Because $F_{ij} = -F_{ji}$, if we set two of the indices equal in this equation, the LHS is identically zero. For example if $i = j$:

$$(0.4) \quad \partial_i F_{ik} + \partial_k F_{ii} + \partial_i F_{ki} = -\partial_i F_{ki} + 0 + \partial_i F_{ki}$$

$$(0.5) \quad = 0$$

Since the three terms are cyclic permutations of each other, setting any other pair of indices equal gives the same result.

If we choose $i = y, j = x, k = z$, we get

$$(0.6) \quad -\partial_y B_y - \partial_z B_z - \partial_x B_x = 0$$

This is the law $\nabla \cdot \mathbf{B} = 0$.

If we now choose $i = y, j = x, k = t$, we get

$$(0.7) \quad -\partial_y E_x + \partial_t B_z + \partial_x E_y = 0$$

This is the z component of Faraday's law $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$.
Other choices give the remaining components of Faraday's law.

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