ELECTROMAGNETIC FIELD TENSOR: A COUPLE OF MAXWELL'S EQUATIONS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 6; Problem 6.10, 7.2.

The electromagnetic field tensor F^{ij} is

$$F^{ij}=\left[\begin{array}{cccc} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{array}\right] \tag{1}$$

The various electromagnetic laws can be expressed by the equation

$$\partial_i F_{jk}+\partial_k F_{ij}+\partial_j F_{ki}=0 \tag{2}$$

The lowered version F_{ij} of F^{ij} in the flat metric of special relativity is

$$F_{ij}=\left[\begin{array}{cccc} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{array}\right] \tag{3}$$

Because $F_{ij}=-F_{ji}$, if we set two of the indices equal in this equation, the LHS is identically zero. For example if $i=j$:

$$\partial_i F_{ik}+\partial_k F_{ii}+\partial_i F_{ki} = -\partial_i F_{ki}+0+\partial_i F_{ki} \tag{4}$$

$$= 0 \tag{5}$$

Since the three terms are cyclic permutations of each other, setting any other pair of indices equal gives the same result.

If we choose $i=y$, $j=x$, $k=z$, we get

$$-\partial_y B_y-\partial_z B_z-\partial_x B_x=0 \tag{6}$$

This is the law $\nabla\cdot\mathbf{B}=0$.

If we now choose $i=y$, $j=x$, $k=t$, we get

$$-\partial_y E_x + \partial_t B_z + \partial_x E_y = 0 \tag{7}$$

This is the z component of Faraday's law $\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$.
Other choices give the remaining components of Faraday's law.

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