ELECTROMAGNETIC FIELD TENSOR: CYCLIC DERIVATIVE RELATION

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 7; Problem 7.6.

We've used the following relation between the derivatives of the electromagnetic field tensor F^{ij} to get several of Maxwell's equations.

$$\partial_i F_{jk} + \partial_k F_{ij} + \partial_j F_{ki} = 0 \tag{0.1}$$

Here we verify that this relation is true when F^{ij} is written in terms of the four-potential, that is

$$F^{ij} = \partial^i A^j - \partial^j A^i \tag{0.2}$$

We can lower both indices in this equation and the plug it into the first equation:

$$\begin{aligned}
&\partial_i F_{jk} + \partial_k F_{ij} + \partial_j F_{ki} = \partial_i \left(\partial_j A_k - \partial_k A_j\right) + \partial_k \left(\partial_i A_j - \partial_j A_i\right) + \partial_j \left(\partial_k A_i - \partial_i A_k\right) && (0.3)\\
&\qquad = \partial_i \partial_j A_k - \partial_j \partial_i A_k + \partial_k \partial_i A_j - \partial_i \partial_k A_j + \partial_j \partial_k A_i - \partial_k \partial_j A_i && (0.4)\\
&\qquad = 0 && (0.5)
\end{aligned}$$

The terms in the second line cancel in pairs since the order of the partials doesn't matter.