# ELECTROMAGNETIC FIELD TENSOR: INVARIANCE OF INNER PRODUCT

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 7; Problem 7.7.

We've worked out earlier the scalar quantity $F_{ij}F^{ij} = 2\left(B^2 - E^2\right)$. We can check this for the case of a Lorentz transformation in flat space-time. We found there that

$$E'_x = E_x \tag{1}$$
$$E'_y = \gamma E_y - \gamma\beta B_z \tag{2}$$
$$E'_z = \gamma E_z + \gamma\beta B_y \tag{3}$$
$$B'_x = B_x \tag{4}$$
$$B'_y = -\gamma\beta E_y + \gamma B_z \tag{5}$$
$$B'_z = -\gamma\beta E_z - \gamma B_y \tag{6}$$

Calculating the invariant in the new system we get

$$B'^2 - E'^2 = B'^2_x + B'^2_y + B'^2_z + E'^2_x + E'^2_y + E'^2_z \tag{7}$$
$$= B_x^2 + (-\gamma\beta E_y + \gamma B_z)^2 + (-\gamma\beta E_z - \gamma B_y)^2 - \tag{8}$$
$$E_x^2 - (\gamma E_y - \gamma\beta B_z)^2 - (\gamma E_z + \gamma\beta B_y)^2 \tag{9}$$
$$= B_x^2 + \left(B_y^2 + B_z^2\right)\gamma^2\left(1-\beta^2\right) - E_x^2 - \left(E_y^2 + E_z^2\right)\gamma^2\left(1-\beta^2\right) \tag{10}$$
$$= B^2 - E^2 \tag{11}$$

since $\gamma = 1/\sqrt{1-\beta^2}$. All the cross terms involving the product of a component of $E$ and one of $B$ cancel out between lines 2/3 and 4.