

ELECTROMAGNETIC FIELD TENSOR: INVARIANCE OF INNER PRODUCT

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 7; Problem 7.7.

We've worked out earlier the scalar quantity $F_{ij}F^{ij} = 2(B^2 - E^2)$. We can check this for the case of a Lorentz transformation in flat space-time. We found there that

$$(0.1) \quad E'_x = E_x$$

$$(0.2) \quad E'_y = \gamma E_y - \gamma\beta B_z$$

$$(0.3) \quad E'_z = \gamma E_z + \gamma\beta B_y$$

$$(0.4) \quad B'_x = B_x$$

$$(0.5) \quad B'_y = -\gamma\beta E_y + \gamma B_z$$

$$(0.6) \quad B'_z = -\gamma\beta E_z - \gamma B_y$$

Calculating the invariant in the new system we get

$$(0.7)$$

$$B'^2 - E'^2 = B_x'^2 + B_y'^2 + B_z'^2 + E_x'^2 + E_y'^2 + E_z'^2$$

$$(0.8) \quad = B_x^2 + (-\gamma\beta E_y + \gamma B_z)^2 + (-\gamma\beta E_z - \gamma B_y)^2 -$$

$$(0.9) \quad E_x^2 - (\gamma E_y - \gamma\beta B_z)^2 - (\gamma E_z + \gamma\beta B_y)^2$$

$$(0.10) \quad = B_x^2 + (B_y^2 + B_z^2) \gamma^2 (1 - \beta^2) - E_x^2 - (E_y^2 + E_z^2) \gamma^2 (1 - \beta^2)$$

$$(0.11) \quad = B^2 - E^2$$

since $\gamma = 1/\sqrt{1 - \beta^2}$. All the cross terms involving the product of a component of E and one of B cancel out between lines 2/3 and 4.