

## ELECTROMAGNETIC FIELD TENSOR: INVARIANCE OF INNER PRODUCT

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 7; Problem 7.7.

We've worked out earlier the scalar quantity  $F_{ij}F^{ij} = 2(B^2 - E^2)$ . We can check this for the case of a Lorentz transformation in flat space-time. We found there that

$$E'_x = E_x \quad (1)$$

$$E'_y = \gamma E_y - \gamma\beta B_z \quad (2)$$

$$E'_z = \gamma E_z + \gamma\beta B_y \quad (3)$$

$$B'_x = B_x \quad (4)$$

$$B'_y = -\gamma\beta E_y + \gamma B_z \quad (5)$$

$$B'_z = -\gamma\beta E_z - \gamma B_y \quad (6)$$

Calculating the invariant in the new system we get

$$B'^2 - E'^2 = B_x'^2 + B_y'^2 + B_z'^2 + E_x'^2 + E_y'^2 + E_z'^2 \quad (7)$$

$$= B_x^2 + (-\gamma\beta E_y + \gamma B_z)^2 + (-\gamma\beta E_z - \gamma B_y)^2 - \quad (8)$$

$$E_x^2 - (\gamma E_y - \gamma\beta B_z)^2 - (\gamma E_z + \gamma\beta B_y)^2 \quad (9)$$

$$= B_x^2 + (B_y^2 + B_z^2) \gamma^2 (1 - \beta^2) - E_x^2 - (E_y^2 + E_z^2) \gamma^2 (1 - \beta^2) \quad (10)$$

$$= B^2 - E^2 \quad (11)$$

since  $\gamma = 1/\sqrt{1 - \beta^2}$ . All the cross terms involving the product of a component of  $E$  and one of  $B$  cancel out between lines 2/3 and 4.