SCHWARZSCHILD METRIC: REDSHIFT OF SIRIUS B

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapters 9; Problem 9.2.

The gravitational redshift due to a spherical mass M measured between the Schwarzschild emission radius r_E and reception radius r_R is

$$\frac{\lambda_R}{\lambda_E}=\sqrt{\frac{1-2GM/r_R}{1-2GM/r_E}} \tag{1}$$

For $r_R\to\infty$, the formula reduces to

$$\frac{\lambda_R}{\lambda_E}=\frac{1}{\sqrt{1-2GM/r_E}} \tag{2}$$

We can test this formula with some real data, obtained from the Hubble telescope on Sirius B, the white dwarf companion of Sirius A, the brightest star in the night sky. Sirius B is believed to have a mass 1.02 times that of the sun and a radius of $r_E=5640$ km. The fractional redshift of its light is 2.68×10^{-4} with an uncertainty of around 6%, so we can see if this is consistent with the predicted value.

Since we're making measurements from the Earth, we can take $r_R=\infty$ and thus use the simplified formula. In relativistic units we have

$$\begin{aligned} GM &= \left(7.41\times10^{-28}\right)\left(1.02\times1.989\times10^{30}\right) && (3)\\ &= 1503\text{ m} && (4)\end{aligned}$$

The fractional redshift is

$$\begin{aligned}\frac{\lambda_R-\lambda_E}{\lambda_E} &= \frac{1}{\sqrt{1-2GM/r_E}}-1 && (5)\\ &= \frac{1}{\sqrt{1-2\times1503/5.64\times10^6}}-1 && (6)\\ &= 2.67\times10^{-4} && (7)\end{aligned}$$

The agreement is thus within the error limits of the measurement.