

## SPHERICAL METRIC: DISTANCE IN 2-D CURVED SPACE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 9; Problem 9.3.

As a (very) simple example of calculating distance in a curved space, we'll consider the metric for the surface of a sphere:

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2 \quad (1)$$

For any given sphere,  $R$  is a constant and represents the radius of the sphere when viewed in three dimensions. In the two-dimensional surface of the sphere, however,  $R$  is just a parameter which governs distances on the surface.

If we start at the north pole, where  $\theta = 0$ , and move along a curve of constant  $\phi$  to a point  $\theta = \theta_0$  then  $d\phi = 0$  and

$$ds^2 = R^2 d\theta^2 \quad (2)$$

$$ds = R d\theta \quad (3)$$

$$s = R \int_0^{\theta_0} d\theta \quad (4)$$

$$= R\theta_0 \quad (5)$$

Similarly, if we restrict the curve to constant  $\theta$ , then  $d\theta = 0$  and if we move between  $\phi = 0$  and  $\phi = \phi_0$ :

$$ds^2 = R^2 \sin^2 \theta d\phi^2 \quad (6)$$

$$s = R(\sin \theta) \phi_0 \quad (7)$$

Although these results might seem obvious, that's because we're used to analyzing this situation in 3-d and we have an 'obvious' interpretation of  $R$ ,  $\theta$  and  $\phi$ . Looking at the problem in 2-d,  $\theta$  and  $\phi$  are the coordinates of a point in curved space, and  $R$  is just a parameter that determines how curved the space is.