SPHERICAL METRIC: DISTANCE IN 2-D CURVED SPACE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 9; Problem 9.3.

As a (very) simple example of calculating distance in a curved space, we'll consider the metric for the surface of a sphere:

$$ds^2 = R^2 d\theta^2 + R^2 \sin^2\theta d\phi^2 \tag{1}$$

For any given sphere, R is a constant and represents the radius of the sphere when viewed in three dimensions. In the two-dimensional surface of the sphere, however, R is just a parameter which governs distances on the surface.

If we start at the north pole, where $\theta = 0$, and move along a curve of constant ϕ to a point $\theta = \theta_0$ then $d\phi = 0$ and

$$\begin{aligned} ds^2 &= R^2 d\theta^2 && (2)\\ ds &= R d\theta && (3)\\ s &= R\int_0^{\theta_0} d\theta && (4)\\ &= R\theta_0 && (5) \end{aligned}$$

Similarly, if we restrict the curve to constant θ, then $d\theta = 0$ and if we move between $\phi = 0$ and $\phi = \phi_0$:

$$\begin{aligned} ds^2 &= R^2 \sin^2\theta d\phi^2 && (6)\\ s &= R(\sin\theta)\phi_0 && (7) \end{aligned}$$

Although these results might seem obvious, that's because we're used to analyzing this situation in 3-d and we have an 'obvious' intepretation of R, θ and ϕ. Looking at the problem in 2-d, θ and ϕ are the coordinates of a point in curved space, and R is just a parameter that determines how curved the space is.