

SPHERICAL METRIC: DISTANCE IN 2-D CURVED SPACE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 9; Problem 9.3.

As a (very) simple example of calculating distance in a curved space, we'll consider the metric for the surface of a sphere:

$$(0.1) \quad ds^2 = R^2 d\theta^2 + R^2 \sin^2 \theta d\phi^2$$

For any given sphere, R is a constant and represents the radius of the sphere when viewed in three dimensions. In the two-dimensional surface of the sphere, however, R is just a parameter which governs distances on the surface.

If we start at the north pole, where $\theta = 0$, and move along a curve of constant ϕ to a point $\theta = \theta_0$ then $d\phi = 0$ and

$$(0.2) \quad ds^2 = R^2 d\theta^2$$

$$(0.3) \quad ds = R d\theta$$

$$(0.4) \quad s = R \int_0^{\theta_0} d\theta$$

$$(0.5) \quad = R\theta_0$$

Similarly, if we restrict the curve to constant θ , then $d\theta = 0$ and if we move between $\phi = 0$ and $\phi = \phi_0$:

$$(0.6) \quad ds^2 = R^2 \sin^2 \theta d\phi^2$$

$$(0.7) \quad s = R(\sin \theta) \phi_0$$

Although these results might seem obvious, that's because we're used to analyzing this situation in 3-d and we have an 'obvious' interpretation of R , θ and ϕ . Looking at the problem in 2-d, θ and ϕ are the coordinates of a point in curved space, and R is just a parameter that determines how curved the space is.