CIRCULAR ORBIT AROUND A SUPERMASSIVE BLACK HOLE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 10; Problem 10.9.

Suppose we are unfortunate enough to be orbiting a supermassive black hole with a mass of 10^6 solar masses. For the sun, $GM = 1.477$ km, so for the black hole, $GM = 1.477\times10^6$ km. The radius of the orbit is $r = 10GM = 1.477\times10^7$ km. For comparison, the average earth-sun distance is 1.5×10^8 km, so this orbit is about 10 times closer to the black hole than Earth is to the sun. A rather frightening prospect.

From its radius, we can work out the energy and angular momentum per unit mass. From the formula for angular momentum as measured by the orbiting object:

$$
\begin{aligned}
l^2 &= \frac{r^2GM}{r-3GM} && (1)\\
&= \frac{100\,(GM)^2}{7} && (2)\\
l &= \frac{10}{\sqrt{7}}GM && (3)\\
&= 5.58\times10^6\text{ km} && (4)
\end{aligned}
$$

The energy is

$$
\begin{aligned}
\tilde{E} &= -\frac{GM}{2r}\left(1-\frac{3GM}{r}\right)^{-1}\left(1-\frac{4GM}{r}\right) && (5)\\
&= -\frac{0.6}{20\times0.7} && (6)\\
&= -0.043 && (7)
\end{aligned}
$$

The period of the orbit as measured by the spacecraft can be found from

$$
\begin{aligned}
l &= r^2\omega & (8)\\
&= \frac{2\pi r^2}{T} & (9)\\
T &= \frac{2\pi r^2}{l} & (10)\\
&= \frac{200\sqrt{7}\pi}{10}GM & (11)\\
&= 166GM & (12)\\
&= 2.455\times 10^8 \text{ km} & (13)\\
&= 821 \text{ s} & (14)\\
&= 13.7 \text{ minutes} & (15)
\end{aligned}
$$