CIRCULAR ORBIT AROUND A SUPERMASSIVE BLACK HOLE

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 10; Problem 10.9.

Suppose we are unfortunate enough to be orbiting a supermassive black hole with a mass of 10^6 solar masses. For the sun, $GM = 1.477$ km, so for the black hole, $GM = 1.477 \times 10^6$ km. The radius of the orbit is $r = 10GM = 1.477 \times 10^7$ km. For comparison, the average earth-sun distance is 1.5×10^8 km, so this orbit is about 10 times closer to the black hole than Earth is to the sun. A rather frightening prospect.

From its radius, we can work out the energy and angular momentum per unit mass. From the formula for angular momentum as measured by the orbiting object:

$$l^2 = \frac{r^2 GM}{r - 3GM} \tag{0.1}$$

$$= \frac{100\,(GM)^2}{7} \tag{0.2}$$

$$l = \frac{10}{\sqrt{7}} GM \tag{0.3}$$

$$= 5.58 \times 10^6 \text{ km} \tag{0.4}$$

The energy is

$$\tilde{E} = -\frac{GM}{2r}\left(1 - \frac{3GM}{r}\right)^{-1}\left(1 - \frac{4GM}{r}\right) \tag{0.5}$$

$$= -\frac{0.6}{20 \times 0.7} \tag{0.6}$$

$$= -0.043 \tag{0.7}$$

The period of the orbit as measured by the spacecraft can be found from

$$l = r^2\omega \tag{0.8}$$

$$= \frac{2\pi r^2}{T} \tag{0.9}$$

$$T = \frac{2\pi r^2}{l} \tag{0.10}$$

$$= \frac{200\sqrt{7}\pi}{10}GM \tag{0.11}$$

$$= 166GM \tag{0.12}$$

$$= 2.455 \times 10^8 \text{ km} \tag{0.13}$$

$$= 821 \text{ s} \tag{0.14}$$

$$= 13.7 \text{ minutes} \tag{0.15}$$