

EMBEDDING A 2-D CURVED SURFACE IN 3-D: THE COSH

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 11; Problem 11.07.

A second example embedding a curved-space 2-d metric in 3-d flat space. This time, our 2-d metric is

$$(1) \quad ds^2 = \cosh^2\left(\frac{r}{R}\right) dr^2 + r^2 d\phi^2$$

Here, the ϕ component is already in the required form, so we can equate this to the cylindrical metric directly:

$$(2) \quad dz^2 + r^2 d\phi^2 + dr^2 = \cosh^2\left(\frac{r}{R}\right) dr^2 + r^2 d\phi^2$$

$$(3) \quad \frac{dz}{dr} = \pm \left[\cosh^2\left(\frac{r}{R}\right) - 1 \right]^{1/2}$$

$$(4) \quad = \pm \sinh\left(\frac{r}{R}\right)$$

This integrates directly to give

$$(5) \quad z = \pm R \cosh\left(\frac{r}{R}\right)$$

The upper lobe of this surface looks like this (there is a lower lobe which is a mirror image of the upper one):

