

RED-SHIFTS AND BLUE-SHIFTS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 12; Problem 12.5.

We've seen that the fact that Schwarzschild time coordinate t and the proper time τ are not the same leads to the gravitational redshift. The relation between the wavelength of a photon at two distances r_R and r_E from a mass M is

$$(1) \quad \frac{\lambda_R}{\lambda_E} = \sqrt{\frac{1 - 2GM/r_R}{1 - 2GM/r_E}}$$

We can derive the same formula from the photon's four-momentum. In an observer's locally flat frame, the energy of any object (including a photon) is

$$(2) \quad E_o = -\mathbf{o}_t \cdot \mathbf{p}$$

since $\mathbf{o}_t = [1, 0, 0, 0]$ in that local frame, and $p^t = E_o$. Using the expressions for these two four-vectors in the global Schwarzschild frame, we get

$$(3) \quad \mathbf{o}'_t = \left[\left(1 - \frac{2GM}{r}\right)^{-1/2}, 0, 0, 0 \right]$$

$$(4) \quad p'^t = E_\infty \left(1 - \frac{2GM}{r}\right)^{-1}$$

where E_∞ is the energy of the photon as measured by an observer at infinity. The scalar product gives

$$(5) \quad E_o = -g_{tt}o_t^t p^t$$

$$(6) \quad = \left(1 - \frac{2GM}{r}\right) \left(1 - \frac{2GM}{r}\right)^{-1/2} E_\infty \left(1 - \frac{2GM}{r}\right)^{-1}$$

$$(7) \quad = \frac{E_\infty}{\sqrt{1 - \frac{2GM}{r}}}$$

That is, the energy of a photon increases the closer it gets to the mass. In terms of the wavelength, we can use the relation from quantum mechanics: $E = h\nu = hc/\lambda = h/\lambda$ (taking $c = 1$ as usual). Thus

$$(8) \quad \lambda_o = \lambda_\infty \sqrt{1 - \frac{2GM}{r}}$$

or, if we take the ratio of the wavelengths at two finite radii as at the start:

$$(9) \quad \frac{\lambda_R}{\lambda_E} = \sqrt{\frac{1 - 2GM/r_R}{1 - 2GM/r_E}}$$

which is the same as the original formula.

Another way of saying this is that an observer far from a mass sees light red-shifted compared to an observer near the mass or, conversely, the near observer sees incoming light blue-shifted compared to an observer far from the mass.