RED-SHIFTS AND BLUE-SHIFTS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 12; Problem 12.5.

We've seen that the fact that Schwarzschild time coordinate t and the proper time τ are not the same leads to the gravitational redshift. The relation between the wavelength of a photon at two distances r_R and r_E from a mass M is

$$\frac{\lambda_R}{\lambda_E}=\sqrt{\frac{1-2GM/r_R}{1-2GM/r_E}} \tag{0.1}$$

We can derive the same formula from the photon's four-momentum. In an observer's locally flat frame, the energy of any object (including a photon) is

$$E_o=-\mathbf{o}_t\cdot\mathbf{p} \tag{0.2}$$

since $\mathbf{o}_t=[1,0,0,0]$ in that local frame, and $p^t=E_o$. Using the expressions for these two four-vectors in the global Schwarzschild frame, we get

$$\mathbf{o}_t^{\prime} = \left[\left(1-\frac{2GM}{r}\right)^{-1/2},0,0,0\right] \tag{0.3}$$

$$p^{\prime t} = E_\infty\left(1-\frac{2GM}{r}\right)^{-1} \tag{0.4}$$

where E_∞ is the energy of the photon as measured by an observer at infinity. The scalar product gives

$$E_o = -g_{tt} o_t^{\prime t} p^{\prime t} \tag{0.5}$$

$$= \left(1 - \frac{2GM}{r}\right)\left(1 - \frac{2GM}{r}\right)^{-1/2} E_\infty \left(1 - \frac{2GM}{r}\right)^{-1} \tag{0.6}$$

$$= \frac{E_\infty}{\sqrt{1 - \frac{2GM}{r}}} \tag{0.7}$$

That is, the energy of a photon increases the closer it gets to the mass. In terms of the wavelength, we can use the relation from quantum mechanics: $E = h\nu = hc/\lambda = h/\lambda$ (taking $c = 1$ as usual). Thus

$$\lambda_o = \lambda_\infty \sqrt{1 - \frac{2GM}{r}} \tag{0.8}$$

or, if we take the ratio of the wavelengths at two finite radii as at the start:

$$\frac{\lambda_R}{\lambda_E} = \sqrt{\frac{1 - 2GM/r_R}{1 - 2GM/r_E}} \tag{0.9}$$

which is the same as the original formula.

Another way of saying this is that an observer far from a mass sees light red-shifted compared to an observer near the mass or, conversely, the near observer sees incoming light blue-shifted compared to an observer far from the mass.