DEFLECTION OF LIGHT BY THE SUN

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 13; Exercise 13.5.1.

We can use the formula for trajectory of a photon as it passes a mass to determine the angle of deflection as viewed by an observer far from the mass. The formula is

$$\frac{1}{r}=\frac{1}{r_c}\left(\sin\phi+\frac{3GM}{2r_c}\left(1+\frac{1}{3}\cos 2\phi\right)\right) \tag{1}$$

When the photon is far from the mass, we can take $r\to\infty$ which occurs when the angle ϕ satisfies

$$\sin\phi_0+\frac{3GM}{2r_c}\left(1+\frac{1}{3}\cos 2\phi_0\right)=0 \tag{2}$$

Since we're taking $\phi=\pi/2$ as the angle of closest approach, we would expect $r\to\infty$ to occur when ϕ_0 is close to zero or π. If we take ϕ_0 close to zero, we can use the small angle approximations $\sin\phi_0\approx\phi_0$ and $\cos\phi_0\approx 1$ to get

$$\phi_0\approx-\frac{2GM}{r_c} \tag{3}$$

The fact that $\phi_0<0$ indicates that the trajectory is bent slightly towards the mass on either side, which is what we'd expect. The total deflection δ is therefore twice this angle so

$$\delta=2\left|\phi_0\right|=\frac{4GM}{r_c} \tag{4}$$

To see what this means, suppose we consider the experiment that was done in 1919 to test Einstein's prediction. During the total solar eclipse on 29 May 1919, the positions of several stars near the solar disk were measured. Because the path of a photon that passes very near to the sun is bent towards the sun, it will appear to have moved a small distance *away* from the solar disk compared to where it would be if the sun were not there. (If you find this hard to see you can try a similar experiment at home. Take a piece of wire and bend it slightly around some round object such as an orange and then hold it so that you are looking at the wire end-on. If you sight along the bent end nearest your eye and project this line out to infinity, you should see that the point onto which this line projects is further from the orange than if you sighted along a straight wire that is tangent to the orange at the same point.)

The effect for an object such as the sun is not large. $GM = 1.477$ km and the radius of the sun is 6.955×10^5 km so the deflection angle is $\delta = 8.495 \times 10^{-6}$ radians, which works out to 1.752 arc seconds. The experimental values from the 1919 expeditions ranged between 1.61 and 1.98 arc seconds, so it was taken as a confirmation of Einstein's prediction.

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