MACHOS AND SEEING DISTANT OBJECTS WITH A GRAVITATIONAL LENS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 13; Problem P13.6.

In gravitational lensing, the formula for the relative brightness of the images is given by

$$\frac{I_{\pm}}{I_s}=\frac{1}{4}\left(\frac{\sqrt{\beta^2+4\theta_E^2}}{\beta}+\frac{\beta}{\sqrt{\beta^2+4\theta_E^2}}\pm 2\right) \tag{0.1}$$

where $I_{\pm}$ are the brightnesses of the lensed images and I_s is the brightness of the unlensed source. We can write this as

$$\frac{I_{\pm}}{I_s}=\frac{1}{4}\left(\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{1/2}+\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{-1/2}\pm 2\right) \tag{0.2}$$

The combined brightness ratio of the two images is then

$$\begin{aligned}\frac{I_{tot}}{I_s} &= \frac{I_+}{I_s}+\frac{I_-}{I_s} && (0.3)\\ &= \frac{1}{2}\left(\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{1/2}+\left(1+4\frac{\theta_E^2}{\beta^2}\right)^{-1/2}\right) && (0.4)\end{aligned}$$

The two angles are given by

$$\theta_{\pm}=\frac{\beta}{2}\left(1\pm\sqrt{1+4\frac{\theta_E^2}{\beta^2}}\right) \tag{0.5}$$

so in cases where the two images are so close together they can't be distinguished, we see only the combined light intensity. The distance between the images is

$$\theta_+ - \theta_- = \beta\sqrt{1+4\frac{\theta_E^2}{\beta^2}} = \sqrt{\beta^2+4\theta_E^2} \tag{0.6}$$

so if both β and θ_E are very small, the images will be indistinguishable. From the formula

$$\theta_E \equiv \sqrt{D_{LS}\frac{4GM}{D_L D_S}} \tag{0.7}$$

we see that θ_E will be small if S is much further away than L so that $D_{LS} \approx D_S$ but the distance to the lens still satisfies $D_L \gg 4GM$. The requirement that β is small means merely that S is not too far off being directly behind L.

Since θ_E is a constant for any given S and L, the total intensity depends only on β. From the formula above, we can show (by standard calculus) that $I_{tot}/I_s \geq 1$ for all values of β, so the lensing effect actually produces a brighter image than would be seen without the lens. The effect actually has a practical application in the detection of faint objects. If a lens moves transversely across the sky and passes nearly in front of a dim background source, the source will become brighter as the lens passes across it. Such lens objects are known as *massive compact halo objects* or MACHOs. To see how this works, suppose a MACHO moves at a constant angular speed so that its path across the sky as seen from earth is a straight line. We define β as the angle between the direction to the MACHO and the direction to the distant source, as usual. Let β_0 be the angle of closest approach, that is, the minimum of β as the MACHO passes near the source. Finally, we let α be the angle between the current position of the MACHO and its position when $\beta = \beta_0$. To a good approximation for small angles, the three angles β, β_0 and α form a right triangle, with β the hypotenuse, so

$$\beta^2 = \beta_0^2 + \alpha^2 \tag{0.8}$$

The angle α also represents the position along the path followed by the MACHO, so assuming a constant transverse motion, we have

$$\frac{d\alpha}{dt} = v_\alpha \tag{0.9}$$

for some constant v_α. If we let t_E be the time required for α to change by an angle θ_E, then

$$\frac{d\alpha}{dt} = \frac{\theta_E}{t_E} = v_\alpha \tag{0.10}$$

Therefore, if we define time $t = 0$ to be when $\beta = \beta_0$:

$$\beta^2 = \beta_0^2 + \left(\frac{\theta_E t}{t_E}\right)^2 \tag{0.11}$$

$$= \theta_E^2 \left[\frac{\beta_0^2}{\theta_E^2} + \frac{t^2}{t_E^2}\right] \tag{0.12}$$

$$\equiv \theta_E^2 \left[u_0^2 + \frac{t^2}{t_E^2}\right] \tag{0.13}$$

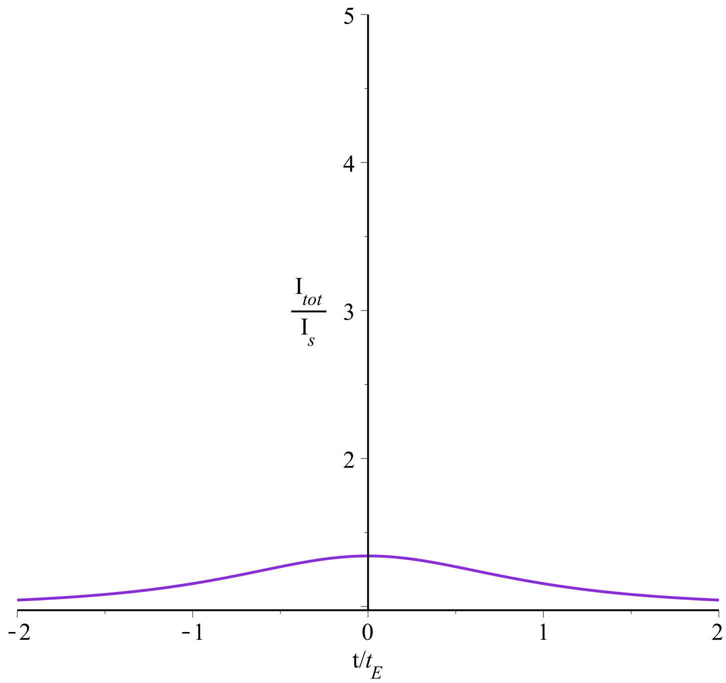
Plugging this back into the equation above, we get

$$\frac{I_{tot}}{I_s} = \frac{1}{2}\left(q(t) + \frac{1}{q(t)}\right) \tag{0.14}$$

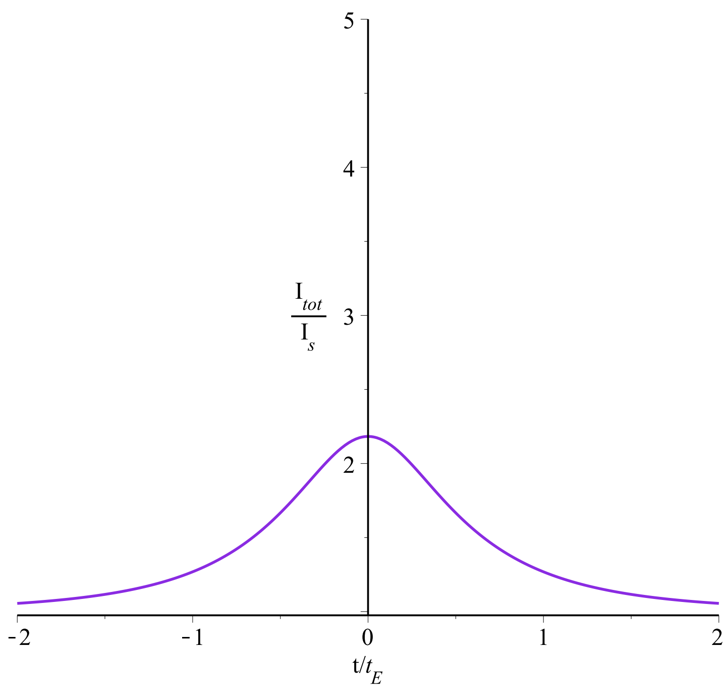
where

$$q(t) \equiv \sqrt{1 + \frac{4}{u_0^2 + t^2/t_E^2}} \tag{0.15}$$

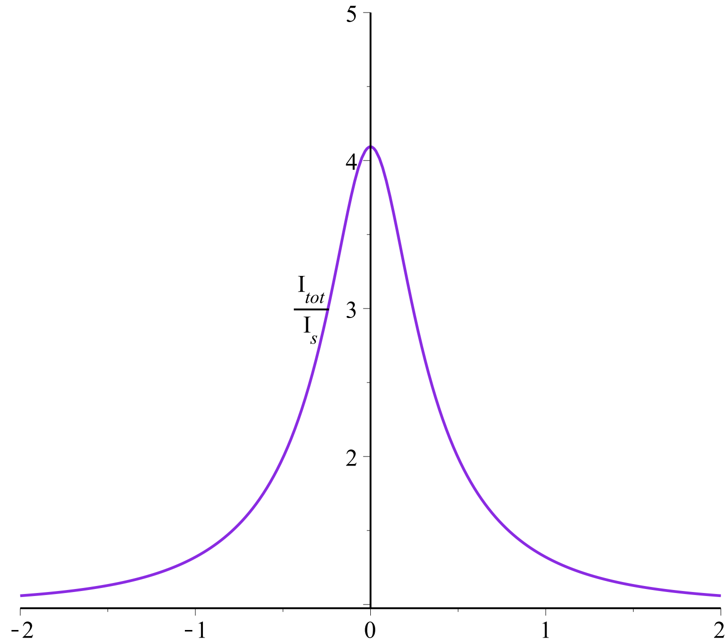
Here are a few plots of I_{tot}/I_s:



u_0=1

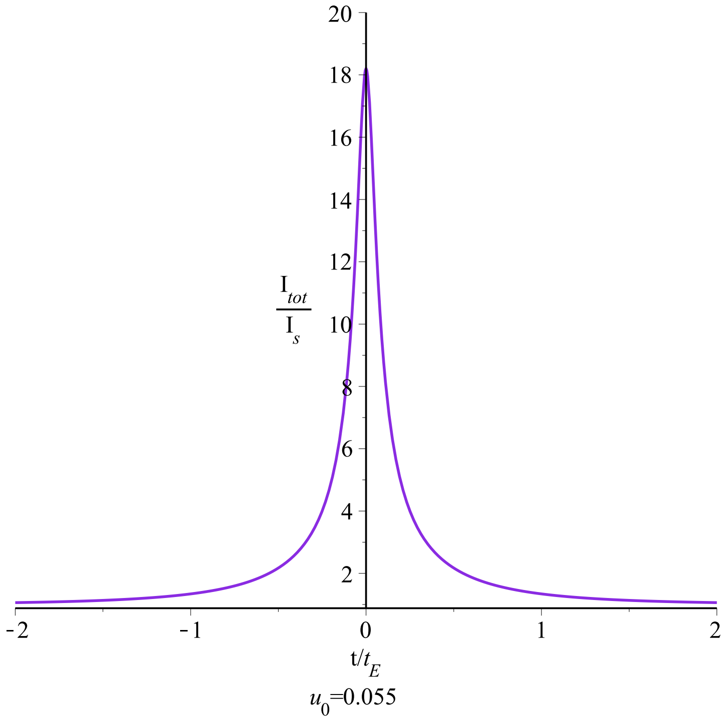


u_0=0.5



We see that the smaller $u_0 = \beta_0/\theta_E$, the higher the peak brightness. For $u_0 = 0.25$, for example, the peak brightness is more than 4 times the unlensed image, so the effect is quite noticeable, and can be used to detect objects that would otherwise be invisible to earthbound telescopes.

A reasonable fit to Fig. 13.6 in Moore's book occurs when $u_0 = 0.055$:



To estimate t_E, we can observe the points at which the curve has the value $I_{tot}/I_s = 2$. In the plot from the formula, this happens at roughly $t/t_E = \pm 0.5$. In Fig 13.6, it happens at around $t = 568$ days and $t = 578$ days so $\Delta t = 10$ days and $\Delta t/t_E = 1$. Thus $t_E = 10$ days.

PINGBACKS

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