

THE SUN AS A GRAVITATIONAL LENS

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 13; Problem P13.9.

One exotic possibility is that of using the sun as a gravitational lens. In order to do this, we would need to place a satellite at an appropriate distance from the sun so that the images of the background source are not hidden by the sun's disk. In this system, the distance from the satellite to the sun is D_L so the angle subtended by the radius R_s of the sun is

$$\theta_s = \frac{R_s}{D_L} \quad (1)$$

In order for the image of the background source to be visible, we must require that the Einstein ring angle θ_E be larger than θ_s , thus the minimum distance D_L is found from

$$\theta_s = \theta_E \quad (2)$$

$$\frac{R_s}{D_L} = \sqrt{D_{LS} \frac{4GM}{D_L D_S}} \quad (3)$$

Since any source will be essentially infinitely far away, we can take $D_{LS} = D_S$ and we get

$$D_L = \frac{R_s^2}{4GM} = 8.2 \times 10^{10} \text{ km} = 548 \text{ AU} \quad (4)$$

At this distance $\theta_E = 8.49 \times 10^{-6}$ radians = θ_s .

If a planet orbits Alpha Centauri at a distance of 1 AU, then since the distance of Alpha Centauri is 4.3 ly = 2.72×10^5 AU, the angle δ between the star and the planet is $1/2.72 \times 10^5 = 3.68 \times 10^{-6}$ radians, or a bit less than half θ_E .

The combined intensity of the two images tends to infinity as the angle between the lens and source goes to zero:

$$\frac{I_{tot}}{I_s} = \frac{1}{2} \left(\left(1 + 4 \frac{\theta_E^2}{\beta^2} \right)^{1/2} + \left(1 + 4 \frac{\theta_E^2}{\beta^2} \right)^{-1/2} \right) \quad (5)$$

If the satellite is lined up perfectly, we would in theory get an infinite magnification of a point source on the planet. If the satellite is 1 km out of line, however, the magnification can be worked out by noting that the angle subtended by this 1 km distance as seen from the sun is

$$\beta_1 = \frac{1}{548 \times 1.496 \times 10^8} = 1.22 \times 10^{-11} \text{ rad} \quad (6)$$

To a good approximation, this is also the angle β between the lines from the satellite to the sun and to the planet, so we can plug this into the formula for the intensity ratio above:

$$\frac{I_{tot}}{I_s} = 7 \times 10^5 \quad (7)$$

The star, however, will also be imaged, but since its angular distance from the planet is $\delta = 3.68 \times 10^{-6}$ radians, the value of $\beta_{\alpha C}$ for the star, assuming the satellite is still at its 1 km point, is $\beta_{\alpha C} = \delta + \beta_1 \approx \delta$. Plugging this into the intensity formula we get

$$\frac{I_{\alpha C}}{I_s} = 2.47 \quad (8)$$

Thus the planet will be magnified considerably more than the star, but the difference in inherent brightnesses would probably mean that the star's image would still appear brighter, so some form of shielding would be needed.

The shift in position of the satellite by an angle of β_1 (equivalent to 1 km at its distance of 548 AU) gives rise to a shift at the other end of

$$\beta_1 \times 2.72 \times 10^5 \text{ AU} \times 1.496 \times 10^8 \text{ km} \cdot \text{AU}^{-1} = 496 \text{ km} \quad (9)$$

so the planet doesn't appear as a point object at this resolution, meaning that the full magnification as calculated above wouldn't be reached anyway.