

EVENT HORIZON: TIME AND SPACE SWAP ROUND

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 14; Box 14.3.

When we first looked at the t component of the Schwarzschild metric we noted that for an object at rest, the t component is related to the object's proper time by

$$(0.1) \quad \Delta\tau = \sqrt{1 - \frac{2GM}{r}} \Delta t$$

If the object is at the event horizon, that is, $r = 2GM$, then $\Delta\tau = 0$ and since $ds^2 = -d\tau^2$, this means that $ds^2 = 0$ for the object, no matter what Δt is. A zero space-time interval can exist only for photons (or other massless particles), so what happens if a massive particle approaches $r = 2GM$? The only way we can reconcile the Schwarzschild metric with an object at the event horizon is if the object is not at rest (remember the assumption that the object *was* at rest led to the zero space-time interval, so that assumption must be wrong). In other words, as an object approaches the event horizon, it is compelled to keep moving; there is no way it can stop itself from continuing past the event horizon.

The explanation of this phenomenon goes like this. In any metric, a time-like interval is always represented by a value of $ds^2 < 0$. In the Schwarzschild metric:

$$(0.2) \quad ds^2 = - \left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

this means that the two events that define the interval must always be separated by a non-zero interval in the coordinate t . It is possible for all the other intervals (the space intervals dr , $d\theta$ and $d\phi$, for $r > 2GM$) to be zero, that is, it's possible for the two events to occur at the same place, but they must always be separated by a non-zero time interval.

If we carry the Schwarzschild metric through the event horizon so that $r < 2GM$, then the signs of the dt^2 and dr^2 components flip, so that $-\left(1 - \frac{2GM}{r}\right) dt^2 > 0$ and $\left(1 - \frac{2GM}{r}\right)^{-1} dr^2 < 0$. Thus a timelike interval now means that dr

must be non-zero, rather than dt . In other words, the physical meanings of r and t have swapped; r now plays the role of a time component and t of a space component.

Inside the event horizon, we can write the metric as

$$(0.3) \quad ds^2 = - \left(\frac{2GM}{r} - 1 \right)^{-1} dr^2 + \left(\frac{2GM}{r} - 1 \right) dt^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

The proper time for an object at rest is now

$$(0.4) \quad d\tau^2 = -ds^2 = \left(\frac{2GM}{r} - 1 \right)^{-1} dr^2 - \left(\frac{2GM}{r} - 1 \right) dt^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

Since $\frac{2GM}{r} - 1 > 0$ inside the event horizon, the maximum proper time interval occurs when $dt = d\theta = d\phi = 0$, in other words, for a purely radial path. Since a geodesic is the path of longest proper time, the geodesic is a purely radial path through space-time. For a path from $r = 0$ to $r = 2GM$, this time interval is given by the same formula we evaluated in the last post, with $R = 2GM$

$$(0.5) \quad \Delta\tau = \frac{\pi R^{3/2}}{\sqrt{8GM}} = \pi GM$$

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