EVENT HORIZON: TIME AND SPACE SWAP ROUND

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Reference: Moore, Thomas A., *A General Relativity Workbook*, University Science Books (2013) - Chapter 14; Box 14.3.

When we first looked at the t component of the Schwarzschild metric we noted that for an object at rest, the t component is related to the object's proper time by

$$\Delta\tau=\sqrt{1-\frac{2GM}{r}}\Delta t \tag{1}$$

If the object is at the event horizon, that is, $r=2GM$, then $\Delta\tau=0$ and since $ds^2=-d\tau^2$, this means that $ds^2=0$ for the object, no matter what Δt is. A zero space-time interval can exist only for photons (or other massless particles), so what happens if a massive particle approaches $r=2GM$? The only way we can reconcile the Schwarzschild metric with an object at the event horizon is if the object is not at rest (remember the assumption that the object *was* at rest led to the zero space-time interval, so that assumption must be wrong). In other words, as an object approaches the event horizon, it is compelled to keep moving; there is no way it can stop itself from continuing past the event horizon.

The explanation of this phenomenon goes like this. In any metric, a timelike interval is always represented by a value of $ds^2<0$. In the Schwarzschild metric:

$$ds^2=-\left(1-\frac{2GM}{r}\right)dt^2+\left(1-\frac{2GM}{r}\right)^{-1}dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2 \tag{2}$$

this means that the two events that define the interval must always be separated by a non-zero interval in the coordinate t. It is possible for all the other intervals (the space intervals dr, $d\theta$ and $d\phi$, for $r>2GM$) to be zero, that is, it's possible for the two events to occur at the same place, but they must always be separated by a non-zero time interval.

If we carry the Schwarzschild metric through the event horizon so that $r<2GM$, then the signs of the dt^2 and dr^2 components flip, so that $-\left(1-\frac{2GM}{r}\right)dt^2>0$ and $\left(1-\frac{2GM}{r}\right)^{-1}dr^2<0$. Thus a timelike interval now means that dr

must be non-zero, rather than dt. In other words, the physical meanings of r and t have swapped; r now plays the role of a time component and t of a space component.

Inside the event horizon, we can write the metric as

$$ds^2 = -\left(\frac{2GM}{r}-1\right)^{-1}dr^2+\left(\frac{2GM}{r}-1\right)dt^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2 \tag{3}$$

The proper time for an object at rest is now

$$d\tau^2 = -ds^2 = \left(\frac{2GM}{r}-1\right)^{-1}dr^2-\left(\frac{2GM}{r}-1\right)dt^2-r^2d\theta^2-r^2\sin^2\theta d\phi^2 \tag{4}$$

Since $\frac{2GM}{r}-1>0$ inside the event horizon, the maximum proper time interval occurs when $dt=d\theta=d\phi=0$, in other words, for a purely radial path. Since a geodesic is the path of longest proper time, the geodesic is a purely radial path through space-time. For a path from $r=0$ to $r=2GM$, this time interval is given by the same formula we evaluated in the last post, with $R=2GM$

$$\Delta\tau = \frac{\pi R^{3/2}}{\sqrt{8GM}} = \pi GM \tag{5}$$

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